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Adverse Selection with the Boot on the Other Foot: Insurer Insolvency as a Problem in Asymmetric Information

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Abstract: The problem of insurer insolvency has almost exclusively been seen as an issue in regulation. However, it is also clear that there is an obvious element of asymmetric information present when insolvency is possible. Insurers are clearly better informed of their probability of defaulting on an insurance arrangement than are their insureds, and just as clearly, that probability affects the value of the contract to the insured. With that in mind, we recast the issue of insurer insolvency within the context of asymmetric information, specifically, adverse selection. In our model, the (risk-neutral) insurer is the informed agent, and the (risk-averse) policyholder is the uninformed principal. Thereby, the classic player identities in an asymmetric information problem are reversed. We find equilibrium contract menus for the cases of perfect competition between insurers, and for the case of a single monopolistic insurer.

Keywords: insurance insolvency, asymmetric information, adverse selection

JEL Classifications: D80, D82

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1. Introduction

The insurance market is a typical example of a market that is subject to asymmetric information. A standard assumption is that when an insurance arrangement is being considered, insurers are not able to make decisions with sufficient information on the risks presented by the potential policyholder. However, more in general information will be asymmetrically distributed across both insurers and policyholders. Asymmetries in the possession of information will inevitably lead to inefficiencies in the operation of the insurance market. In this paper, in contrast to almost all of the existing literature on asymmetric information in insurance, we consider an interesting case in which the insurer has more information than does the insured. The information in question is the likelihood of insolvency of the insurer.

The issue of solvency is traditionally dealt with directly by regulation rather than considering it to be a problem in asymmetric information. However, whether solvency regulation can induce insurance companies to ensure their solvency when they have better information on their financial situation than would regulators or insurance consumers is an issue which has received little attention in the literature. Ashby (2011) studied the causes of the banking crisis and the shortcomings of current solvency regulation, and raised doubts about the direction of certain insurance regulatory reforms, such as concerns about capital requirements and quantitative risk assessment. In addition, the research emphasized the importance of the human element and the accuracy of the information. Mahul & Wright (2004) provided empirical evidence of the impact of solvency regulation on the number of insurance companies and the frequency of insolvencies. The minimum capital requirement does reduce the number of bankruptcies of insurance companies, but this can only be achieved by preventing relatively risky and small companies from entering the insurance market, something that is likely impossible to achieve when insurer risk is unobservable. The minimum capital limit has no effect on the frequency of bankruptcies of companies that have already entered the insurance market. Indeed, there is little evidence that any of the several forms of solvency regulation has a significant deterrent effect on insolvency.

Obviously, the currently existing solvency regulations cannot completely eliminate the negative impact of asymmetric information on the insurance industry. The accuracy of the information and the monitoring of adverse selection and moral hazard are issues that insurers need to pay attention to. Rees et al. (1999) took asymmetric information regarding solvency into account and argued that regulation is not needed so long as full information is available. If consumers are fully informed about the risks of insurance companies' default risk, solvency regulation is unnecessary since under those conditions the company always provides sufficient capital to ensure solvency, unless there are restrictions on the composition of its asset portfolio. In this case, Rees et al. (1999) conclude that the purpose of solvency regulation should be only to provide consumers with the information needed.

Currently, practically all of the existing research on the equilibrium of the insurance market under asymmetric information is focused on the situation of the policyholder being the informed agent and the insurer being the uninformed principle. The problem of an insurance market in which the policyholder is the uninformed principle and the insurer is the informed agent seems to have been largely ignored to date in the literature. Such an asymmetry in information in the insurance market will, logically, give rise to the two main problems, namely adverse selection and moral hazard, but where the identities of the informed and uninformed participants are reversed from the standard theory. The concept of insurer insolvency, however, presents exactly that sort of environment, and this is what motivates the present article. The objective is to consider a market in which policyholders desire insurance coverage for a loss, but in which there is some probability of insolvency of the insurer. Crucially, the insurer is informed of the probability of insolvency, and the policyholder is not.[2] We believe that the present article is the first in the literature to contemplate this particular problem, from the perspective of the economics of asymmetric information. The problem will be tackled under the assumption that there is no effective solvency regulation that would perfectly overcome the asymmetric information, which is the general message from the existing insurance solvency literature. The research here will provide a new perspective on the solution of asymmetric information problems in the insurance market by finding incentive-compatible separating equilibrium contract menus for the cases that we study.

The idea of asymmetric information regarding insurer insolvency leads to a few interesting departures from the standard way in which such problems are analysed. When the insurer has

[2] An insurance company's capital is fluid, and will change over time according to the actual operating conditions of the insurance company. Although relevant agencies require insurance companies to disclose their capital, it cannot be open and transparent at every single period, when investments mature in an uncertain way, and claim payments also present continuous stochastic changes in reserves.

private information about their risk of insolvency, then the insurer becomes the "informed agent" and the insurance purchaser becomes the "uninformed principal", thus reversing the standard roles. However, when insolvency is brought into an insurance model, there are more than two states of nature, as opposed to the standard two-dimensional analysis of asymmetric information in insurance, in which there is only a loss state and a no-loss state. Thus, our analysis cannot be carried out in the standard two-dimensional state contingent claims graph. Further, when the insurance consumer is the informed agent, then standardly the solution to asymmetric information issues such as adverse selection resides in the insured accepting more risk in order to reveal their type. However, in our setting, if we retain the standard assumption that the insurer is risk-neutral (as we will), then using risk to reveal type will not work. This then begs the question of what, if any, is the equilibrium contract menu for the type of situation that we are interested in resolving.

2. Game set-up and comparison with the standard insurance model under adverse selection

In this paper, we only consider the issue of adverse selection – the informed agent (here the insurer) is allocated a "type", which cannot be modified, and which is only observable by that agent and, in particular, not by the uninformed principal (the policyholder). An agent's type is defined by their probability of insolvency over the life of the insurance contract that is the underlying element of the model. The game works as follows:

1. There is a population of potential policyholders (principals), who are all identical in respect of their utility function (assumed to be risk averse), their initial wealth, and their loss distribution (which is assumed to be a fixed possible loss, that occurs with a known probability). These principals seek to insure their risky wealth situation with a second-party insurer.
2. Nature allocates to each insurer (agent) a probability of insolvency, s_i, where $i \in (L, H)$, and where $0 < s_L < s_H < 1$. An agent's type is therefore either L or H.
3. Each agent observes their type, and then offers an insurance contract to the market.
4. Each principal then chooses a contract (or chooses to remain uninsured).
5. An equilibrium occurs when no agent (insurer) would like to change the contract they offer, given the contracts offered by all other agents.

There are a few similarities and a few important differences between the game we study and the “standard model” of insurance under adverse selection, where by “standard model” we refer explicitly to the Rothschild and Stiglitz (1976) model in which the potential policyholders are the informed agents, and the insurers are the uninformed principals. To make these things clear, the following table spells out the most important of these differences and similarities.

Table 1: Comparison with the standard model of insurance under adverse selection

	Standard model	**Insolvency model**
Who are the informed agents?	Potential policyholders	Insurers
Who are the uninformed principals?	Insurers	Potential policyholders
How many states of nature?	Two	Four
Assumed risk aversions	Agents risk averse, principals risk neutral	Agents risk neutral, principals risk averse
Who moves first?	Uninformed principals	Informed agents
How many different types?	Two	Two
How many contracts will eventuate? How many will be used?	Normally two contracts will eventuate, one offered for each type, and normally both will be used	Two will be normally be offered, but only one will be used (since there is only one type of policyholder)

Perhaps the most important differences are (i) the number of different states of nature, which will require a different graphical analysis, (ii) the change in risk aversion, which will imply that self-selection in our setting cannot rely on adding risk to an agent’s situation as is the case in the standard model, and (iii) the fact that the informed agent moves first rather than the uninformed principal, which changes the model from one of self-selection to one of signaling.

3. The contract space environment

By introducing the possibility of insolvency into a model of insurance, at the minimum it is necessary to consider 4 different states of nature; loss and solvency, loss and insolvency, no loss and solvency, and no loss and insolvency. This cannot be represented graphically in a traditional 2-dimensional state contingent claims graph, as it would require four different axes to measure the four different states of nature. For that reason, here we follow Wilson (1977) and use a graph that shows the insurance contract variables on the axes – contracted indemnity (I) on the horizontal axis, and contracted premium paid (Q) on the vertical. Such a "contract space" graph can depict situations of more than two states of nature, and therefore it is the setting that we use to study insurance insolvency under asymmetric information.

The origin of the graph, $(Q, I) = (0,0)$, is the default of no insurance contract. The indifference curves of a wealth-loving risk-averse insurance consumer passing through this contract space are positively sloped concave curves, with greater preference towards the south-east in the graph.[3] The indifference curves of a risk-neutral insurer (iso-expected profit curves) are positively sloped straight lines, with greater preference towards the north-west. It can easily be verified that, at the origin, the policyholder's indifference curve is steeper than the insurer's iso-profit line. This immediately implies that mutually beneficial trade is possible. Indeed, under an assumption of perfectly competitive insurance markets, were it not for asymmetric information, each policyholder would search for the contract along the relevant expected profit equal to 0 line such that a tangency occurs with their own indifference curve.

4. Exogenous risk of insolvency

4.1 Introducing potential insolvency

In the standard insurance model, insurers are assumed to be exogenously endowed with sufficient capital such that insolvency does not occur. However, in fact insurers do face the problem of insolvency caused by such things as insufficient capital and an unexpectedly high run of claims. The size of the capital of an insurer is one of the important factors affecting the probability of insolvency. In a paper that focusses on insurance insolvency (contract non-performance), but where the information structure is similar to the classic Rothschild-Stiglitz setting, Mirma & Wambach (2019) assume that before entering the market, insurers decide on the amount of up-front money they will use as reserve funds. Insufficient initial capital creates an endogenous insolvency risk since, depending on contract offers and the distribution of risk types over the contracts of an insurance company, there may not be enough assets to cover all claims. In Mirma & Wambach's model, policyholders can observe the initial capital of the insurer, so theirs is not an asymmetric information model of the same sort as that which is

[3] These two affirmations can easily be checked using the implicit function theorem. Concavity, of course, relies upon the assumption of strictly risk-averse preferences.

explored in the model of the present article.

In the present research, we focus on the probability of an insurer becoming insolvent, under the assumption that, when initiating their contract, policyholders cannot observe an insurer's capital, their investment strategies, or their existing set of potential contractual claim obligations. In considering the probability of insolvency, we apply the same concept from Mirma & Wambach (2019), that is, the probability of insolvency is determined, in a large part, by the initial reserves of the insurance company being small in relation to their existing set of potential claim obligations.[4] The main contribution of our article is to consider the principal-agent setting for insurance under the assumption that it is the insurer who is informed (of their probability of insolvency) and the insured policyholder who is uninformed. In order to focus attention on the issue of the information asymmetry regarding insolvency, unlike traditional models, we dispense entirely with the assumption of different policyholder risk profiles. Concretely, here the probability of loss occurring of the policyholder is assumed to be common knowledge, known by the insurance company. In effect, there is only one "type" of (representative) policyholder, with known probability p of a loss of known size L.

4.2 Assumptions and baseline model

First of all, we consider a simple baseline model with full information. Assume there is only one "representative" policyholder. The policyholder has initial wealth of w, but suffers a loss of value L with probability p. The policyholder can purchase insurance against that risk. An insurance contract is a set of two numbers, $C = (Q, I)$, where Q is the premium charged for insurance and I is the indemnity to be paid if the insured accident happens. Naturally, we restrict the set of contracts to satisfy $I > Q$. The insurer has a known initial reserve fund $R > 0$, but there is an exogenous (to the contract under analysis here) risk under which R goes to 0 with probability s ($s > 0$), and R has no loss with probability $1 - s$.[5] Crucially, we assume here that the insurer's risk regarding R cannot be insured.[6] Solvency for the duration of the contract is expressed by $R \geq I - Q$ for all feasible I and Q that can be chosen. Since in any

[4] This could be because any investments of the reserves could turn out lower than expected, or the size of claims of other contracts could be higher than expected.

[5] The risk is exogenous in that it is independent of the policyholder's contract. The risk could be the outcome of an investment of the reserve funds, or perhaps the outcome of the set of all other policyholders' contracts. It is only in order to ease the calculations that the risk is assumed to be an all-or-nothing outcome for R. In a more general model, the final value of R could be set as a continuous random variable.

[6] That assumption would, of course, be relatively simple to dispense with. But doing so would simply shift the problem of insolvency to the primary insurer, given that the re-insurer could suffer some risk of insolvency.

contract $I > Q$, if R goes to 0 all the insurance company has left to finance claims is the premium income collected, Q. In such a case, if the insured accident happens, the insurance company becomes insolvent and pays back Q, defaulting on the rest of the contracted indemnity.

Assume that insurers are risk-neutral and there are two types of insurance company in the market with s_L and s_H ($s_L < s_H$), respectively. This probability cannot be observed by the policyholders. Each insurer is identical in all except s. We focus here on the problem of adverse selection, in which although each insurer is informed of their own value of probability of insolvency, s, they cannot alter that probability or credibly communicate it to the policyholder. Since there is a single type of policyholder, each insurer type will only offer a single contract which is either a type-L contract C_L or a type-H contract C_H, depending on the type of insurer offering the contract.

To begin with, it is necessary to investigate the indifference curves of an insurer and of a policyholder in the contract space graph.

The expected final wealth of a type-*i* insurer offering contract $\mathrm{C} = (Q, I)$ is

$$E_i B(Q, I) = (1 - s_i)\big(p(R + Q - I) + (1 - p)(R + Q)\big) + s_i(p \times 0 + (1 - p)Q)$$

$$= (1 - s_i)(R + Q - pI) + s_i(1 - p)Q$$

$$= (1 - s_i)(R - pI) + (1 - s_i p)Q$$

The first-derivative of the insurer's expected final wealth function with respect to the premium Q and the indemnity I are;

$$\frac{\partial E_i B}{\partial Q} = 1 - s_i p > 0$$

$$\frac{\partial E_i B}{\partial I} = -(1 - s_i)p < 0$$

Using these two equations, it can be obtained from the implicit function theorem that the indifference curve of the insurance company in contract space is linear, with strictly positive slope;

$$\left.\frac{\partial Q}{\partial I}\right|_{E_i B} = \frac{(1 - s_i)p}{1 - s_i p} > 0$$

Further, the indifference curve is less steep the higher is the probability of insolvency, as can be directly seen by deriving the slope with respect to s_i;

$$\frac{d}{ds_i}\frac{(1-s_i)p}{1-s_ip} = \frac{-p(1-p)}{(1-s_ip)^2} < 0$$

Therefore, the higher is s_i, the flatter is the insurer's indifference curve. Figure 1 plots the indifference curves for the two different types of insurer assuming no contract, $(Q,I) = (0,0)$. As in the model without insolvency risk, the insurer (regardless of type) has higher expected profit this higher is the premium and the lower is the indemnity, that is, the insurer's preference is towards the north-west in the graph.

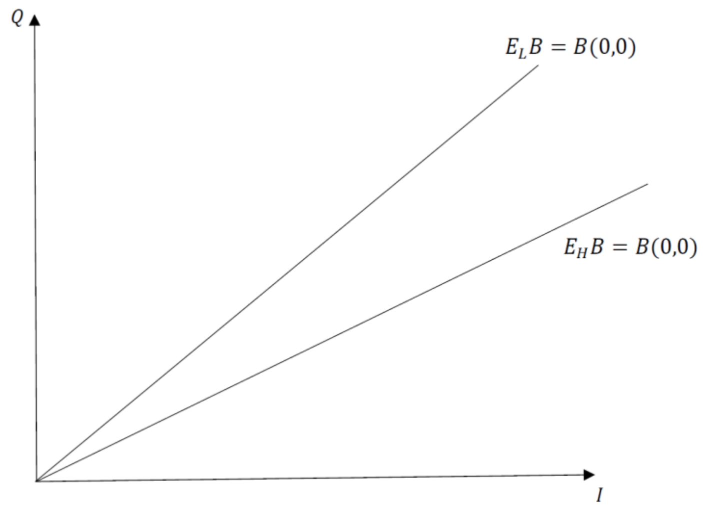

Figure 1: The reservation indifference curves for two different types of insurer

Next, consider the policyholder's indifference curves. The policyholder's expected utility after purchasing the contract (Q,I) from a type-i insurer is

$$E_i u(Q,I) = (1-p)u(w-Q) + p[(1-s_i)u(w-Q-L+I) + s_i u(w-L)]$$

The two first-derivatives of this are:

$$\frac{\partial E_i u}{\partial Q} = -(1-p)u'(w-Q) - p(1-s_i)u'(w-L-Q+I) < 0$$

$$\frac{\partial E_i u}{\partial I} = p(1-s_i)u'(w-L-Q+I) > 0$$

Using these equations, it can be obtained from the implicit function theorem that the policyholder's indifference curves have positive slope with preferences towards the south-east in the graph. Specifically, the slope of the policyholder's indifference curve is

$$\left.\frac{\partial Q}{\partial I}\right|_{E_i u} = \frac{p(1-s_i)u'(w-Q-L+I)}{(1-p)u'(w-Q)+p(1-s_i)u'(w-Q-L+I)} > 0$$

Divide the top and bottom of above formula by $u'(w-Q-L+I)$, to get

$$\left.\frac{\partial Q}{\partial I}\right|_{E_i u} = \frac{p(1-s_i)}{(1-p)\left(\frac{u'(w-Q)}{u'(w-Q-L+I)}\right)+p(1-s_i)}$$

which is strictly positive and less than 1 at all contracts. Specifically, at the default contract of $Q = I = 0$, the slope of the policyholder's indifference curve when she contracts with a type-i insurer is

$$\left.\frac{\partial Q}{\partial I}\right|_{E_i u} = \frac{p(1-s_i)}{(1-p)\left(\frac{u'(w)}{u'(w-L)}\right)+p(1-s_i)}$$

However, notice that under the assumption of strict risk aversion, $\frac{u'(w)}{u'(w-L)} < 1$, so

$$\left.\frac{\partial Q}{\partial I}\right|_{E_i u} = \frac{p(1-s_i)}{(1-p)\left(\frac{u'(w)}{u'(w-L)}\right)+p(1-s_i)} > \frac{p(1-s_i)}{(1-p)+p(1-s_i)} = \frac{p(1-s_i)}{1-ps_i}$$

That is, at the origin of the graph, the policyholder's indifference curve is steeper than the insurer's indifference curve. This immediately indicates that there are mutual gains from trade, when the policyholder is informed of the type of insurer she is dealing with.

In the scenario without the risk of insolvency it is a simple matter to show that, under risk aversion, the policyholder's indifference curve is concave. The same holds true in the exogenous risk case that is under study here, and for completeness we specifically prove this to be the case.

Lemma 1: So long as the policyholder's utility function is strictly concave in wealth, the indifference curves in the exogenous risk environment are concave.

Proof: To prove the lemma, we show that the expected utility function with a type-i insurer is strictly concave in the vector (Q, C). Then, since any concave function is quasi-concave, we know that associated with any given level of expected utility is a strictly convex better set, $U =$

$\{(Q, I): E_i u(Q, I) \geq c\}$. The upper frontier of any such better set is an indifference curve, and thus it must be a concave curve. To that end, consider the four second-derivatives of expected utility;

$$\frac{\partial^2 E_i u}{\partial Q^2} = (1-p)u''(w-Q) + p(1-s)u''(w-L-Q+I) < 0$$

$$\frac{\partial^2 E_i u}{\partial I^2} = p(1-s)u''(w-L-Q+I) < 0$$

$$\frac{\partial^2 E_i u}{\partial Q \partial I} = \frac{\partial^2 E_i u}{\partial I \partial Q} = -p(1-s)u''(w-L-Q+I) > 0$$

The function $E_i u(Q, I)$ is strictly concave in the vector (Q, I) if the following hold:

$$\frac{\partial^2 E_i u}{\partial Q^2} < 0, \quad \frac{\partial^2 E_i u}{\partial I^2} < 0$$

$$\frac{\partial^2 E_i u}{\partial Q^2} \times \frac{\partial^2 E_i u}{\partial I^2} - \frac{\partial^2 E_i u}{\partial Q \partial I} \times \frac{\partial^2 E_i u}{\partial I \partial Q} > 0$$

Clearly, the first two hold (expected utility is concave in Q and in I individually) under the assumption of strict concavity of the utility function. Now consider the third inequality. We have

$$\frac{\partial^2 E_i u}{\partial Q^2} \times \frac{\partial^2 E_i u}{\partial I^2}$$

$$= [(1-p)u''(w-Q) + p(1-s)u''(w-L-Q+I)] \times p(1-s)u''(w-L-Q+I)$$

But this is just

$$(1-p)u''(w-Q)p(1-s)u''(w-L-Q+I) + [p(1-s)u''(w-L-Q+I)]^2$$

On the other hand,

$$\frac{\partial^2 E_i u}{\partial Q \partial I} \times \frac{\partial^2 E_i u}{\partial I \partial Q} = [p(1-s)u''(w-L-Q+I)]^2$$

Therefore, we do indeed have

$$\frac{\partial^2 E_i u}{\partial Q^2} \times \frac{\partial^2 E_i u}{\partial I^2} - \frac{\partial^2 E_i u}{\partial Q \partial I} \times \frac{\partial^2 E_i u}{\partial I \partial Q} = (1-p)u''(w-Q)p(1-s)u''(w-L-Q+I) > 0$$

Which completes the proof that expected utility is strictly concave in the contract.

QED

In the case of asymmetric information, the policyholder prefers the southeast direction in the contract space, while the insurer prefers the northwest direction in the contract space. Figure 2 shows policyholders' and insurers' preferences.

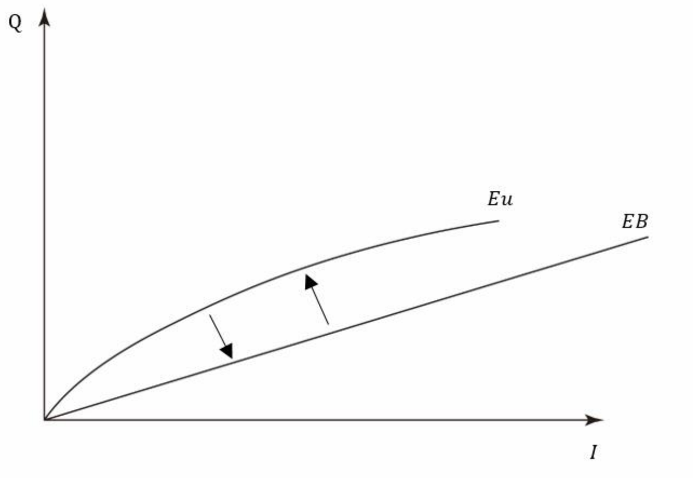

Figure 2: Policyholders' and insurance companies' preferences in the contract space.

4.3 Perfect competition with symmetric information

Under perfect competition, a type-i insurer's final wealth from any given contract must be equal to what it gets from no contract.

$$E_i B(Q, I) = (1 - s_i)R = B(0,0)$$

While with a contract, the insurer's final wealth will be

$$E_i B(Q, I) = (1 - s_i)(R - pI) + (1 - s_i p)Q$$

Therefore, under perfect competition among insurers, (Q, I) must satisfy

$$Q(1 - s_i p) = (1 - s_i)pI$$

$$Q^* = \frac{(1 - s_i)p}{1 - s_i p} L$$

This is a decreasing function of s_i.

Then, the tangency condition for an optimal contract is

$$\frac{(1-s_i)p}{1-s_ip}=\frac{p(1-s_i)}{(1-p)\left(\frac{u'(w-Q)}{u'(w-Q-L+I)}\right)+p(1-s_i)}$$

Simplifying the above equation, we get

$$1-p=\frac{(1-p)u'(w-Q)}{u'(w-Q-L+I)}$$

Therefore, we can infer that the optimal coverage at the tangency point is full coverage, independent of the probability of insolvency:

$$I^*=L$$

At the tangency point between the insurer's indifference curve and the policyholder's indifference curves, the policyholder gets full coverage with premium $Q^*=\frac{(1-s_i)p}{1-s_ip}L$. If $s_i=0$ (the insurer is always solvent), the policyholder gets full coverage at a fair premium pL.

Of course, it is well-known that a risk-averse policyholder who gets insurance at a "fair" price $Q^*=pL$ will choose to be fully insured. However, this conclusion is arrived at under the implicit assumption that the insurer will always be able to perform the contract; that is, the insurer will always have sufficient solvency. Obviously, this changes when the insurance company has a risk of defaulting on the indemnity payment. Doherty and Schlesinger (1990) focus on the purchasing of an insurance policy when there is a chance of default. In their model, as opposed to ours that follows, consumers are fully informed about the insurance premium and the possibility of default of the insurer. They show that an increase in the probability of insolvency does not necessarily reduce optimal coverage. This is consistent with our findings. As has just been shown, under full information with solvency risk, the optimal premium $Q^*=\frac{(1-s_i)p}{1-s_ip}L$ takes the insurer's insolvency risk into account, so that optimal coverage $I^*=L$ is not affected by the insolvency probability s. All of the adjustments for solvency risk are in the premium, not the coverage.

The policyholder's final wealth at the tangency point is

$$E_iW(Q^*,I^*)=w-pL-Q^*(1-ps_i)+p(1-s_i)I^*$$

Substituting $Q^* = \frac{(1-s_i)p}{1-s_ip}L$ and $I^* = L$ into this equation gives

$$E_iW(Q^*, I^*) = w - pL$$

Therefore, with the tangency contract (Q^*, I^*), the policyholder gets full coverage and retains the uninsured expected value of wealth. Figure 3 shows the equilibrium which there is only one type of insurer in the market. $C^* = (Q^*, I^*)$ is the optimal contract.

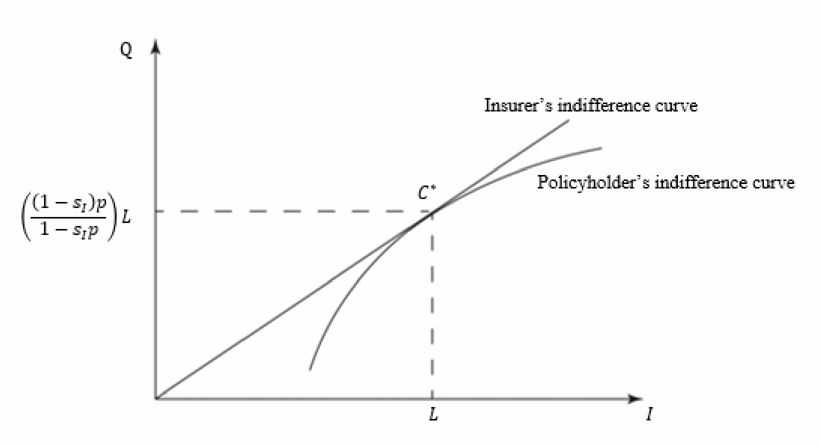

Figure 3: Full information equilibrium when there is one type of insurer in the market

Given that $I^* = L$ is independent of s_i, and $Q^* = \frac{(1-s_i)p}{1-s_ip}L$ is a decreasing function of s_i, we can draw the full information equilibrium when there are two types of insurers with s_L and s_H, respectively. This is done in Figure 4.

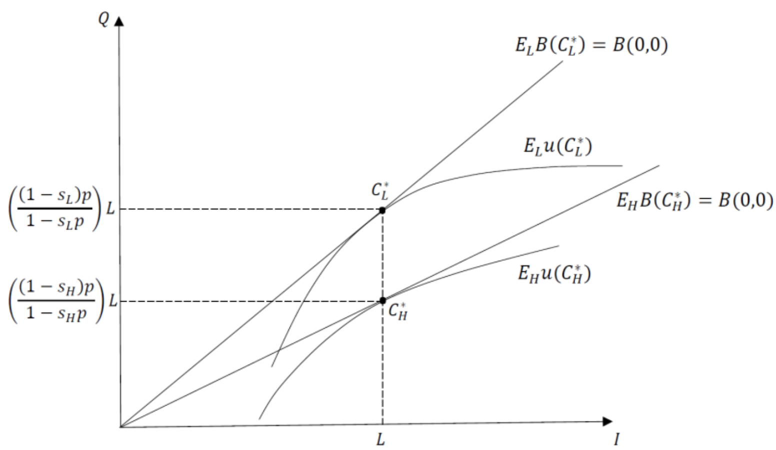

Figure 4: Symmetric information equilibrium when there are two types of insurer in the market

If s_i is observed by the policyholder, then an equilibrium under full information will be achieved. As shown in Figure 4, the indifference curves of policyholders are tangent to the insurer's reservation indifference curves $E_L B = 0$ and $E_H B = 0$ at C_L^* and C_H^*, respectively. Obviously, the policyholder needs to pay higher premiums to the type-L insurers than to type-H insurers, and get the same full coverage indemnity L.

Of course, in a situation in which there are only two types of insurer and perfect information, since all insurance consumers are identical, they would likely all strictly prefer to deal with the same one of the two insurers. Effectively, there would only be room in the market for one of the two insurers. The answer to which insurer will survive is set out in the following result.

Result 1: In the full information setting under perfect competition, all policyholders will purchase only from type-L insurers.

Proof: The expected utility of a policyholder who purchases a full information contract from a type-i insurer is

$$E_i u(Q_i^*, I_i^*) = (1-p)u(w - Q_i^*) + p[(1-s_i)u(w - Q_i^* - L + I_i^*) + s_i u(w - L)]$$

Since we know that $I_i^* = L$ for both $i = L, H$, this becomes

$$E_i u(Q_i^*) = (1-p)u(w - Q_i^*) + p[(1-s_i)u(w - Q_i^*) + s_i u(w - L)]$$

$$= (1 - ps_i)u(w - Q_i^*) + ps_i u(w - L)$$

Consider the first derivative of this with respect to s_i :

$$\frac{\partial E_i u(Q_i^*)}{\partial s_i} = p\big(u(w - L) - u(w - Q_i^*)\big)$$

However, since $u(w - L) < u(w - Q_i^*)$ due to the premium on a contract being less than the value of the total loss, it happens that the effect of a higher probability of insolvency upon expected utility is negative. Finally, since $s_H > s_L$, the type-L contract is preferred in a full information situation.

QED

In the full information setting, then, all policyholders will choose to insure with a type-L insurer. The negative effect of a higher premium is outweighed by the positive effect of a lower probability of insolvency.

4.4 Perfect competition with asymmetric information

The basic model in which we shall analyze asymmetric information is the principal-agent model, but in which the insurance company is the agent and the policyholder is the principal. That is, in all of our analysis it is the insurance company that is fully informed, and the policyholder who is not. In general, a set of contracts is in equilibrium if no insurance company has an incentive to change its contract offer given the contracts of all other insurers. An equilibrium is when each contract generates nonnegative profits and there is no alternative set of contracts that earns positive profits in aggregate and nonnegative profits individually.

If, instead of the symmetric information setting assumed above, insurer type is assumed to be unobservable, then it is clear that the symmetric information equilibrium in which only the low-risk insurer can survive in the market is no longer valid. A type-H insurer can increase their expected profit by passing themselves off as type-L, since they prefer C_L^* to C_H^*. Under adverse selection, any contract above $E_H B = 0$ is preferred to any contract on $E_H B = 0$ by a type-H insurer.

To begin with, we can show that in the case of perfect competition in the insurance market with adverse selection regarding the probability of insurer solvency, there cannot be a pooling equilibrium.

Lemma 2: In a situation of perfect competition in the insurance market, there is no option for a pooling equilibrium, in which both types of insurer offer the same contract.

Proof: This is quite easy to see, so we only offer here an outline of the proof. In order for a pooling equilibrium to exist, there would need to be a single contract, say C^*, offered simultaneously by all insurers, and such that no insurer would prefer to unilaterally deviate from that contract. So, assume that such a point exists, and that it is sufficiently attractive to policyholders that they prefer it over no insurance. In order for both types of insurer to participate at C^*, it must be located on or above the reservation iso-expected profit line of type-L insurers, and therefore also above the reservation iso-expected profit line of type-H insurers as well. At that contract then, all insurers share equally in the market, and each type earns the same excess expected profit. Therefore, given that all insurers are offering C^*, each one of them would want to undercut the contract slightly (i.e. move it in a south-east direction) in benefit of all policyholders, thereby benefitting by capturing all of the market, rather than just their equal share. Since all insurers would deviate from a pooling contract, such a situation is never available as an equilibrium.

QED

Now consider the candidate separating equilibrium contract pair shown in Figure 5, where the type-L contract is the origin which is $C_L^* = (0,0)$.

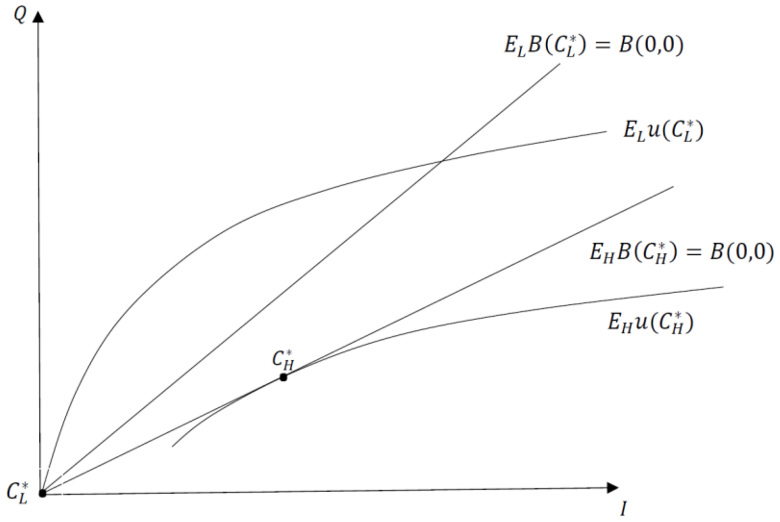

Figure 5: Equilibrium contracts for the case of perfectly competitive insurers

The contract menu $C_L^* = (0,0)$ and C_H^* satisfies incentive compatibility - neither insurance company type (strictly) prefers the contract of the other. It also satisfies participation since each insurance company type is exactly indifferent between participating or not.

Result 2: The contract menu $C_L^* = (0,0)$ and C_H^* constitute the unique separating equilibrium in the market.

Proof: Start with the type-H contract. No insurer can offer anything better to the policyholders, since that would imply moving below the reservation expected profit line. And no single type-H insurer can offer anything above that line, since it would not be taken up by any policyholder given that at the original type-H contract, which is by assumption being offered by all other type-H insurers, policyholder utility is maximized. Therefore, no type-H insurer can do any better than C_H^* given that all other type-H insurers are offering C_H^*. Second, is there any incentive for a type-L insurer to offer anything different than the origin? They cannot offer anything below their reservation iso-expected profit line, since that would imply negative expected profit. And offering anything on or above their iso-expected profit line, with the exception of the origin, would set up a pooling situation because any such offering would attract all type-H insurers away from contract C_H^*. Since lemma 2 rules out any such situation as an equilibrium, there is no possible profitable deviation for type-L insurers from the contract at the origin of the graph. Hence, there is nothing better that either type of insurer could offer, given what the others are offering, and thus the two contracts presented in Figure 5 constitute the unique equilibrium under perfect competition.

QED

Interestingly, this equilibrium involves the low-risk contract being the null contract (no premium, no indemnity), which is not preferred by any insurance consumer to the high risk contract, C_H^*. Therefore, the standard pure adverse selection result holds - high risk types drive out the low risk types, that is, any positive coverage insurance contract is with a high risk insurance company. Notice that this is exactly the opposite market outcome in terms of which insurers survive in the market (and indeed, with exactly the same contract) as what we showed in Result 1 occurs under full information. This therefore means that the information costs of the asymmetric information in this equilibrium are borne by policyholders, since they end up having to insure at the (less preferred under full information) type-H contract.

We state this as a concrete result as follows:

Result 3: In a scenario in which insurers function in a perfectly competitive environment, the final market outcome is exactly the opposite as that which is achieved under full information - only the type-H contract will be used by policyholders, that is, all type-L insurers will drop out of the market.

4.5 Monopolistic insurer with asymmetric information

Next, we turn to the case of a market in which a single insurer operates. Compared with perfect competition, things become more interesting when the market is a monopoly. Figure 6 shows the full information equilibrium condition when the monopolistic insurer could be type-L or type-H. The insurer's objective is to maximize its expected profit.

Indeed, since we know from the previous problem (perfect competition) that it is always possible for either of the two different types of insurer to offer an incentive compatible contract that gives themselves their reservation expected profit, we can in fact ignore the participation constraint of the insurer.

In Figure 6 below, $E_L u = u(0,0)$ and $E_H u = u(0,0)$ are the reservation utility indifference curves of policyholders buying a contract from a type-L and a type-H insurer, respectively. The reservation utility indifference curves both pass through the origin. On these two curves, the utility of the policyholder from buying insurance from a type-i insurer, $E_i u(Q, I)$, is the same as the utility of not buying insurance, $u(0,0)$:

$$E_i u(Q, I) = (1 - p)u(w - Q) + p[(1 - s_i)u(w - Q - L + I) + s_i u(w - L)]$$

$$= (1 - p)u(w) + pu(w - L) = u(0,0)$$

$E_L B(C_L^*)$ and $E_H B(C_H^*)$ are, respectively, the iso-profit lines of a type-L and a type-H insurer, passing through their respective full information optimal contracts; C_L^* and C_H^*. In order to have above reservation expected profits for both types of insurers, we simply shift each insurer type's iso-profit line upward compared to the perfect competition setting, until tangency is reached with the respective reservation indifference curve of the policyholder. The general shape of the iso-profit lines and the policyholder's indifference curves do not change at all from what we already saw above. The full information equilibrium is shown in Figure 6. C_L^* and C_H^* constitute the optimal contract menu under a situation of full information.

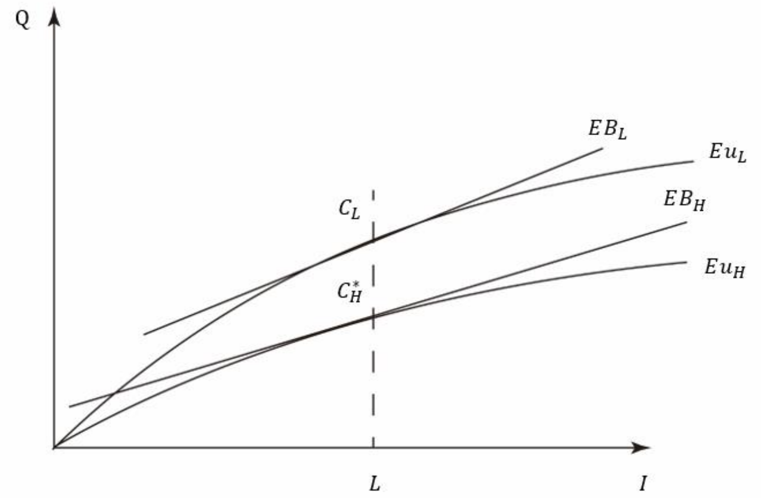

Figure 6: Full information contracts when a monopolistic insurer could be either of two types

Notice that in the full information equilibrium, the following hold true:

1. Regardless of type, the insurer earns expected profit/ strictly greater than reservation level; $E_i B(C_i^*) > B(0,0)$ for $i = L, H$.
2. The policyholder earns reservation utility regardless of the insurer's type; $E_i u(C_i^*) = u(0,0)$ for $i = L, H$.

When we compare the full information equilibrium contracts with a scenario of adverse selection in which the insurer's type (either type-L or type-H) is not observable by the policyholder, the following are evident:

1. Contract C_L^* cannot be offered, since policyholders know that it would be preferred to C_H^* by a type-H insurer, and thus would offer less than reservation utility in expectation.
2. Contract C_H^* can be offered (it is incentive compatible and satisfies participation of the policyholder and a type-H insurer), and so it will be part of any equilibrium contract menu.

To consider what the equilibrium contract menu is, we first need to find the optimal incentive compatible type-L contract C_L^{**} that satisfies participation of both the policyholder and a type-L insurer, given the type-H contract at C_H^*. To that end, Lemma 3 characterises that contract.

Lemma 3: Given a type-H contract at C_H^*, the expected profit maximising type-L contract C_L^{**} binds the participation constraint of a type-L agent and the incentive compatibility constraint of a type-H agent.

Proof: Focus on the following conditions, which uniquely locate the desired contract C_L^{**};

1) It must satisfy the participation constraint of the policyholder; $E_L u(C_L^{**}) \geq u(0,0)$. Graphically, C_L^{**} must lie on or below the policyholder's reservation indifference curve $E_L u = u(0,0)$.
2) It must satisfy participation of a type-L insurer; $E_L B(C_L^{**}) \geq B(0,0)$. Graphically, it must lie on or above the reservation iso-expected profit line of a type-L insurer (which of course passes through the origin of the graph).
3) It must satisfy incentive compatibility; $E_L B(C_L^{**}) \geq E_L B(C_H^*)$ and $E_H B(C_H^*) \geq E_H B(C_L^{**})$. Graphically, the iso-expected profit line of a type-L insurer passing through C_L^{**} must pass above or through C_H^*, and the iso-expected profit line of a type-H insurer passing through C_H^* must pass above or through C_L^{**}.
4) It must lie on the highest possible iso-profit line of a type-L insurer.

We can easily find the contract in question, by simply re-constructing our graph little-by-little. Start by drawing in the reservation utility curve of a policyholder who has a contract with a type-L insurer. This is simply the indifference curve labelled $E_L u = u(0,0)$ in Figure 6 above. Any point on or under that curve will satisfy the policyholder's participation condition, expressed above as condition 1. Then, add to the picture the iso-expected profit line of a type-H insurer passing through the type-H optimal contract C_H^* identified in Figure 6 above. The contract we are searching for must lie on or below this line in order to satisfy incentive compatibility of a type-H insurer, identified above in condition 3. Thus, the optimal type-L contract that we are searching for must lie in the space defined beneath both of these two curves simultaneously, as is shown as a feasible set F in Figure 7.

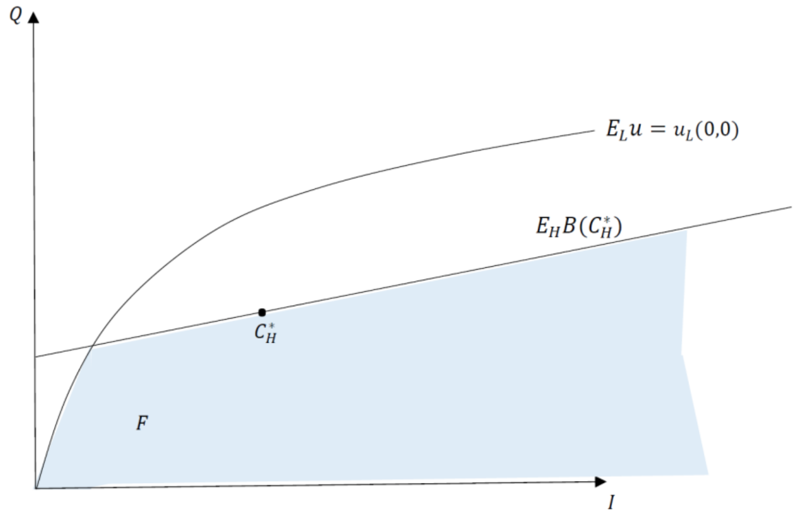

Figure 7: The feasible set for the type-L optimal contract

Now, consider the set of iso-expected profit lies of a type-L insurer. We know that they are steeper than those of a type-H insurer, and they represent higher values of expected profit of the type-L insurer the higher they are in the graph. We also know that they are tangent to the curve $E_L u = u(0,0)$ precisely at the point that is directly vertically above C_H^*. Given the strict concavity of the policyholder's reservation indifference curve. This implies that any point on the policyholder's reservation indifference curve that is vertically to the left of C_H^*, the indifference curve is steeper than the type-L iso-expected profit line. Start then with the iso-expected profit line $E_L B = B(0,0)$, which indicates the reservation expected profit of a type-L insurer. It passes through the origin of the graph, and it intersects $E_H B(C_H^*)$ at some point (it is irrelevant whether that intersection is above or below C_H^*). This is shown in Figure 8, where the intersection point is labelled A.

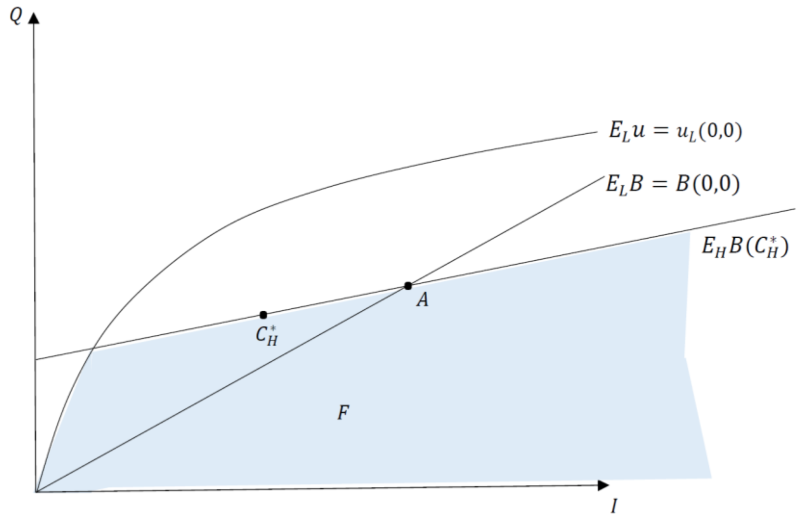

Figure 8: Point A, the starting point for the search for the type-L optimal contract

Now, all that is left to do is to move the type-L iso-profit line upwards in successive parallel movements, until it cannot be moved any further upwards without leaving the feasible set F. It is clear that this process uniquely identifies C_L^{**} as the unique expected profit maximising type-L contract, given the type-H contact at C_H^*, as indicated in Figure 9.

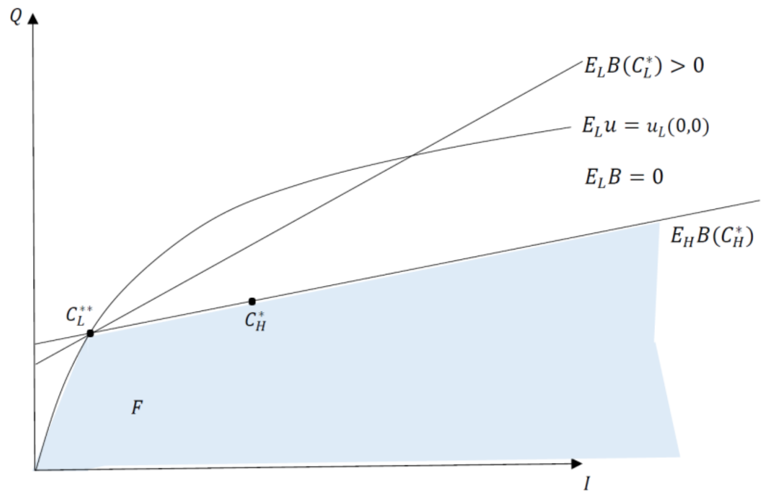

Figure 9: Location of optimal type-L contract

QED

Figure 9 shows the candidate for the separating equilibrium when there is a single insurer which could be either of two types. The relevant type-L contract is the point of intersection between the reservation indifference curve of the policyholder when contracting with a type-L insurer and the iso-expected profit line of a type-H insurer at their optimal full information contract. Notice that this contract menu is incentive compatible, since a type-L insurer prefers C_L^{**} over

C_H^*, and a type-H insurer (weakly) prefers C_H^* over C_L^{**}. Of course this menu also satisfies participation of both insurer types[7] and the policyholder. The policyholder is indifferent between the two contracts, since both provide reservation utility. In this candidate for equilibrium contract menu, the high-risk contract has full coverage at a high premium, while the low-risk contract has partial coverage at a low premium.

The candidate contract menu in Figure 9 is an equilibrium if the following two conditions are met;

1. When the insurer turns out to be type-L, there is nothing better that they can offer than C_L^{**}, given that if they were type-H their offer would be C_H^*.
2. When the insurer turns out to be type-H, there is nothing better that they can offer than C_H^*, given that if they were type-L their offer would be C_L^{**}.

Result 4: In a scenario in which there is a single insurer, whose type (either type-L or type-H) is not observable by the policyholder, then the unique equilibrium contract menu is separating, and involves a type-H insurer offering the full information type-H contract, and a type-L insurer offering a contract that binds the type-H incentive compatibility constraint and the policyholder's participation constraint.

Proof: Start with contract C_L^{**}, given contract C_H^* for a type-H insurer. In what we have just shown above, C_L^{**} maximizes a type-L insurer's expected profit, at the same time as guaranteeing participation by both the insurer and the policyholder, and respecting incentive compatibility with respect to contract C_H^*. Therefore, C_L^{**} is indeed a best response to C_H^*. Next, consider C_H^*, given contract C_L^{**} for a type-L insurer. A type-H insurer cannot offer any contract above the iso-expected profit line passing through C_H^*, as it would not be acceptable to any policyholder, given that it would not satisfy their participation constraint (which is satisfied at contract C_L^{**}). And if a type-H insurer attempted to offer a contract below C_H^*, then their expected profit would unambiguously drop relative to C_H^*, therefore there is no incentive to do so. Thus, C_H^* is the best contract a type-H insurer can offer, given that a type-L insurer would offer C_L^{**}, and so the menu (C_L^{**}, C_H^*) constitutes the unique separating equilibrium when the insurer is a monopolist.

[7] Notice that the type-L insurer's iso-expected profit line at the separating equilibrium is unambiguously higher than their reservation iso-expected profit line.

QED

In this equilibrium, policyholders are left indifferent between the two contracts, since each offers them exactly reservation utility. In any case, only one of the two contracts will actually be offered to the market, depending on whether the insurer turns out to be type-L or type-H. They will signal their true type by their contract offer, and all policyholders will purchase insurance at the offered contract. Insurance in this model may therefore lead to the market being completely absorbed by high risk insurance, or by low risk insurance. Indeed, if when nature makes her move at the outset of the game, the insurer is allocated to be type-L with probability t, then with that same probability the market will only contain contract C_L^{**}, while with probability $1-t$, the market will only contain contract C_H^*.

Relative to the situation of full information, the policyholder is left indifferent, as is a type-H insurer, while a type-L insurer suffers a loss in expected profit. As is standard in adverse selection models, the "good" type is made to suffer the efficiency costs implied by the asymmetric information, although now this is done directly in a worse premium-coverage pair rather than a more risky situation.

5. Comparison of equilibria with the standard model of insurance under adverse selection

It is worthwhile to consider how the final equilibria in our model of adverse selection with insurer insolvency compares with the standard model in which the policyholders are the agents and the insurers are principles. This comparison is summarized in the following two tables:

Table 2: Comparison of equilibria between the insolvency model and the standard model under perfect competition

	Standard model	**Insolvency model**
Equilibrium	Separating equilibrium exists subject to sufficient low-risk agents in the model	Separating equilibrium exists
Contract menu	High-risk agent contract is the full information high-risk	High-risk agent contract is the full information high risk

	contract. Low-risk agent contract binds low-risk participation and high-risk incentive compatibility	contract. Low-risk agent contract is not to offer coverage. Low-risk contract binds low risk participation and high-risk incentive compatibility
Contracts used	Both types of contract will be used by policyholders	Only the high-risk contract will be used by policyholders
Comparison with full information setting	High-risk agents and principals are indifferent between adverse selection equilibrium and full information setting. Low-risk agents lose with respect to full information.	All agents are indifferent between adverse selection equilibrium and full information equilibrium. Principals are worse off with adverse selection since their most preferred insurer is forced from the market.

Table 3: Comparison of equilibria between the insolvency model and the standard model with a monopolistic insurance provider

	Standard model	**Insolvency model**
Equilibrium	Equilibrium always exists	Equilibrium always exists
Contract menu	Separating equilibrium. Neither contract is, in principal, at the full information point. Low-risk contract binds the agent's participation constraint and the high-risk agent's incentive compatibility constraint.	Separating equilibrium. The high-risk contract is at the full information point, the low-risk contract is not. Both contracts bind the participation constraints of the two types of principal, and the low-risk contract binds the high-risk agent's incentive compatibility constraint.

Contracts used	Both contracts will be used in the market, since there will be two types of policyholder	Only one contract will be used in the market, depending on the type assigned to the insurer
Comparison with full information setting	High-risk agent is better off than under full information. Low-risk agent is indifferent to full information. The principal loses with respect to full information.	High-risk agent is indifferent with respect to full information. The low-risk agent is worse off. The principals are worse off also.

6. Conclusion

This article analyses the operation of the insurance market when insurers may be of two types, defined by their probability of insolvency, and where an insurer's type is private information to that insurer. This is a standard setting of adverse selection, but where the identities of the "agent" and "principal" are reversed from the standard asymmetric information model of insurance. We find the second-best equilibrium contract menus for the cases in which insurers function in a perfectly competitive market, and in which there is a single monopolistic insurer.

There are certain differences in the equilibrium contract menus when compared to the standard setting in which policyholders are the agents and the insurer is the principal. First, in the perfectly competitive market, the existence of insurer adverse selection results in only type-*H* insurers operating in the market. That is, the well-known "lemons principle" is obtained. This is not the case for the model with policyholder adverse selection. Second, under a situation of a monopolistic insurer, a full separating equilibrium occurs in which the costs of the asymmetric information are borne by the principals (policyholders) and the insurer if she is type-*L*.

Our model is based on a number of simplifying assumptions, any of which may be relaxed in future research on the topic. In reality, the insurance market is much more complex than we assume, so our model and findings are just a starting point. For example, it would be interesting to extend the study to a situation of continuous types, and it would also be interesting to consider the sister setting of moral hazard, when the insurer can take unobserved actions that affect their probability of insolvency. It would also be interesting, although very complex, to consider a situation in which there is asymmetric information on both sides of the market – that is, policyholders can be of more than one type as well as insurers.

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