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Bond Finance and the Leverage Ratio

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Abstract: A binding pledgeable income constraint limits movements in the leverage ratio but permits some flexibility in the choice of bond versus loan finance in response to changes in key parameters. Due to the existence of distress costs of bond finance in the low payoff state, the share of bond finance remains low compared to more expensive loan finance under both constrained and unconstrained profit maximization.

Keywords: Bonds, Loans, Leverage ratio, Distress cost, Pledgeable income constraint

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The Global Financial Crisis and its aftermath have spurred renewed interest in the factors that affect the composition of debt finance between bank loans and bonds.[1] Crouzet (2018) and Darmouni and Papoutsi (2021) propose models where a profit maximizing firm must balance the savings accruing from cheaper bond finance in good times with the additional distress cost of bond finance in bad times. In this paper I focus on the sensitivity of bond finance and the leverage ratio to a changing economic environment. The comparative statics results indicate that the choices made by an optimizing firm can differ sharply, depending critically on whether it faces a binding pledgeable income constraint or not.

1. The Model[2]

The firm maximizes expected profit through investing in risky projects. Investment is financed partly through debt via bank loans or bonds and partly through internal cash holdings A. The firm has to determine the scale of investment I and β, the share of borrowed funds through bond finance. Investment in projects generates two outcomes, a high payoff R >1 with probability p and a low payoff χ with probability $1\text{-}p$. The low payoff has the further characteristic that it is inversely related to the share of bond finance: $= \chi_0 - \chi_1\beta - (\frac{1}{2})\beta^2$. Thus a firm who relies on bond finance is exposed to distress costs in the low pay-off state which further reduce the payoff. Such distress costs arise because firms find it costly to reach agreement on renegotiating existing debt facilities (including rolling over debt) with individual bond holders in bad times. By contrast, a firm relying exclusively on bank loans and its own assets to finance investment can avoid distress costs altogether. The drawback of bank credit is that it is more expensive than bond finance. More specifically, the cost of loans is given by the sum of the lending rate and the positive parameter c: $r_L + c$. The parameter c captures the additional implicit cost of bank-sourced credit through loan covenants or similar restrictive measures, or high capital requirements. The cost of open-market credit is given by r_B and sensitive to the risk appetite of bond holders. Indeed as risk appetite, denoted by $\gamma > 0$, increases, the gap between the bank lending and the bond rate increases: $r_B + \gamma = r_L$.[3]

Two separate cases are of interest. One involves a binding pledgeable income constraint while the other does not. The pledgeable income constraint involves a friction. Only a

[1] The intellectual foundation of this literature was laid by Holmstrom and Tirole (1997). Adrian, Colla and Shin (2013) and Becker and Ivashina (2014) explore the composition of debt finance in the United States using large micro panel data sets.
[2] The description of the model follows Darmouni and Parpoutsi (2021).
[3] Schwert (2020) finds that banks earn a premium relative to bondholders after adjusting for credit risk.

fraction $\theta < 1$of the high payoff can be guaranteed by the firm along with the low payoff to cover the cost of debt finance. The pledgeable income constraint takes the following form:

$$(p\theta R + (1-p)(\chi_0 - \chi_1\beta - (\tfrac{1}{2})\beta^2))I \geq \rho(I - A) \quad (1)$$

$(I - A)$ = borrowed funds

$\rho = \beta r_B + (1-\beta)(r_L + c) = r_B + (1-\beta)(\gamma + c)$ = cost of debt finance.

The Lagrangean can be stated as:

$$\mathcal{L} = (pR + (1-p)(\chi_0 - \chi_1\beta - (\tfrac{1}{2})\beta^2))I - (\beta r_B + (1-\beta)(r_L + c))(I - A) + \lambda(\,(p\theta R + (1-p)(\chi_0 - \chi_1\beta - (\tfrac{1}{2})\beta^2))I - (\beta r_B + (1-\beta)(r_L + c))(I - A) \quad (2)$$

λ = Lagrange multiplier.

1.1 Unconstrained Profit Maximization

If there is no binding constraint, then $\lambda = 0$. The first-order conditions for the choice variables I and β are:

$$(pR + (1-p)(\chi_0 - \chi_1\beta - (\tfrac{1}{2})\beta^2)) - \;(\beta r_B + (1-\beta)(r_L + c)) = 0 \quad (3)$$

$$(1-p)(-\chi_1 - \beta)I + (\gamma + c)(I - A) = 0 \quad (4)$$

Expected return equals the cost per unit of investment and the excess cost of bond finance equal the savings on bond finance at the margin.

Equation (4) reduces to an expression that relates the share of bond finance to the leverage ratio and parameters of the model:

$$\beta^{UC} = \phi^{UC}\left(\frac{\gamma + c}{1-p}\right) - \chi_1 \quad (5)$$

$$\phi^{UC} = \frac{I - A}{I}$$

The share of bond finance is positively related to the share of borrowed funds in total investment, risk appetite, the additional cost of bank loans, and the probability of the high pay-off outcome. It is inversely related to the size of the parameter χ_1 in the cost function for bond finance in the low pay-off state.[4]

1.2 Constrained Profit Maximization

If the pledgeable income constraint is binding, then pledgeable income equals the cost of borrowing funds. With the right-hand side of equation (1) now equalling its left-hand side, the Lagrangean can be restated as

$$\mathcal{L} = pR(1-\theta)I + \lambda(\,(p\theta R + (1-p)(\chi_0 - \chi_1\beta - (\tfrac{1}{2})\beta^2))I - (\beta r_B + (1-\beta)(r_L + c))(I - A) \quad (6)$$

The binding constraint establishes a proportional relationship between the scale of investment and assets A.

$$I = \frac{\left(r^B + (1-\beta)(\gamma + c)\right)A}{r^B + (1-\beta)(\gamma + c) - (p\theta R + (1-p)\left(\chi_0 - \chi_1\beta - \left(\frac{1}{2}\right)\beta^2\right))} \quad (7)$$

The first-order condition for β again establishes a relationship between leverage relative to investment $\frac{I-A}{I} = \phi^C$ where the superscript *c* denotes that the ratio is formed under a binding constraint:

$$\beta^C = \phi^C\left(\frac{\gamma+c}{1-p}\right) - \chi_1. \quad (8)$$

Comparing equation (8) with (5) reveals that the shares of bond finance in the two cases differ only to the extent that the leverage ratios differ. Combining equations (7) and (8) yields the solutions for β^C and ϕ^C which are reported in Table 1.

2. Share of Bond Finance and Leverage

In this section we compare the sensitivity of the leverage ratio (ϕ) and the share of bond finance (β) to changes in key parameters of the model in the two scenarios described in sections 1.1 and 1.2. As a start, we assign the following parameter values to mark a respective benchmark case: $\{r_B \to 1.04, \chi_0 \to 0.25, \chi_1 \to 0.05, c \to 0.01, \gamma \to 0.01, R \to 1.1, p \to 0.95, \theta \to 0.75, A \to 10\}$.

[4] Eq. (5) and Eq.(3) determine the solutions for the share of bonds and the leverage ratio (and also the scale of investment) . The solutions for both β^{UC} and ϕ^{UC} appear in Table 1.

The first column of Table 2 presents the solutions for both β and ϕ under unconstrained and constrained profit maximization in the baseline case. The remaining columns report how the two variables respond to changes in key parameters of the model in both scenarios.

A binding constraint limits changes in the bond share and the leverage ratio. Notice that the leverage ratio in particular changes very little in response to parameter changes. In the unconstrained case the two variables respond more forcefully to changes in the parameters. In the case of a slight increase in χ_1, the sensitivity of the low payoff to the bond share, the optimal response by a firm operating under a binding constraint is to reduce the share of bonds while holding the leverage ratio relatively constant.[5] This implies that almost all of the adjustment is borne by β. Another way to see this is to take the derivative of equation (8) with respect to χ_1:

$$\frac{d\beta^C}{d\chi_1} = \frac{d\phi^C}{d\chi_1}\frac{(\gamma + c)}{1 - p} - 1 \quad < 0 \tag{9}$$

With $\frac{d\phi^C}{d\chi_1} \cong 0$, it follows that $\frac{d\beta^C}{d\chi_1} \cong -1$.

In the absence of a binding constraint the response of the leverage ratio is positive and very large: $\frac{d\phi^{UC}}{d\chi_1} > 0$. In fact it so large that it swamps the negative direct effect, resulting in an increase in the share of bonds in debt finance following an increase in the size of χ_1:[6]

$$\frac{d\beta^{UC}}{d\chi_1} > 0. \tag{10}$$

If risk appetite (γ) increases, the share of bonds increases relative to the baseline case irrespective of whether the firm faces a binding constraint or not. Not surprisingly, an unconstrained firm increases its bond share by more than a constrained firm. The effect of an increase in risk appetite on the leverage ratio is very different. The leverage ratio for an unconstrained firm increases from 0.63 to 0.76 while for a constrained firm it decreases

[5] With ϕ^C changing very little, the scale of investment remains almost invariant, decreasing only slightly. This follows from *A* being constant.
[6] In Table 1, SR^{UC} decreases in size as $\chi_1$increases, resulting in a larger ϕ^{UC}. For β^{UC} to increase, the decrease in SR^{UC} must dominate $d\chi_1(1 - p) > 0$. In the unconstrained case investment is more variable than under a binding constraint.

minimally from 0.753 to 0.752. Again it is helpful to examine the derivative of β with respect to γ:

$$\frac{d\beta^C}{d\gamma} = \frac{d\phi^C}{d\gamma}\frac{(\gamma + c)}{1-p} + \frac{\phi^C}{1-p} \tag{11}$$

With $\frac{d\phi^C}{d\gamma} \cong 0$ only the second term of the derivative matters, and consequently $\frac{d\beta^C}{d\gamma} \cong \frac{\phi^C}{1-p}$. In the unconstrained case, $\frac{d\phi^{UC}}{d\gamma}$ is strictly positive and along with the second term accounts for the greater increase in the share of bond finance relative to the case of a binding constraint. An increase in c produces equivalent results.

The fourth column presents the case of a tighter pledgeable income constraint through a decrease in θ. It results in a lower share of bond finance and the lowest leverage ratio of the four comparative statics exercises.

The final case considers a higher probability of the high payoff outcome. The increase in probability is minimal at 0.001, but this case proves nevertheless intriguing as it leads to vastly different responses in the two variables of interest. Under a binding constraint, the firm increases the share of bond finance but leaves the leverage ratio virtually unchanged. In contrast, an unconstrained firm reduces both β and ϕ.

Consideration of the derivatives in the two scenarios is again very helpful.

$$\frac{d\beta^C}{dp} = \frac{d\phi^C}{dp}\frac{(\gamma + c)}{1-p} + \frac{\phi^C(\gamma + c)}{(1-p)^2} \tag{12}$$

In the constrained case, the leverage ratio moves minimally in response to the increase in probability, resulting in $\frac{d\phi^C}{dp}$ being close to zero. Hence,

$$\frac{d\beta^C}{dp} \cong \frac{\phi^C(\gamma + c)}{(1-p)^2} > 0. \tag{13}$$

A constrained firm increases bond finance in response to an increase in the probability of the high payoff outcome. For an unconstrained firm just the opposite happens. The higher probability of a successful investment outcome induces the firm to substitute from bond finance towards the more lucrative alternative of loan finance that is not encumbered by the distress cost in the low-payoff state. This is expedited by a substantial decrease in the leverage ratio which swamps the positive effect of the second term in the derivative below:

$$\frac{d\beta^{UC}}{dp} = \frac{d\phi^{UC}}{dp}\frac{(\gamma + c)}{1 - p} + \frac{\phi^{UC}(\gamma + c)}{(1 - p)^2} < 0. \tag{14}$$

3. Conclusion

This paper examines the sensitivity of bond and loan finance to changes in the key parameters that characterize the behavior of a representative firm and its operational environment. A binding pledgeable income constraint limits movements in the leverage ratio but permits a little more flexibility in the choice of bond versus loan finance. Due to the existence of distress costs of bond finance in the low payoff state, the share of bond finance remains low compared to loan finance under both constrained and unconstrained profit maximization.

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Table 1: Solutions for the Share of Bond Finance and the Leverage Ratio

	Unconstrained Case	Constrained Case
$\boldsymbol{\beta}$	$\frac{c+\gamma-\chi_1(1-p)-SR^{UC}}{1-p}$	$1+\frac{r_B}{c+\gamma}-\frac{SR^C}{(c+\gamma)}$
$\boldsymbol{\phi}$	$1-\frac{SR^{UC}}{c+\gamma}$	$\frac{(1-p)((c+\gamma)(1+\chi_1)+r_B-SR^C)}{(c+\gamma)^2}$
	$SR^{UC} = \sqrt{(c+\gamma-(1-p)\chi_1)^2+2(1-p)((1-p)\chi_0+pR-(c+\gamma+r_B))}$	$SR^C = \sqrt{(c+\gamma+r_B)^2+2(c+\gamma)(r_B\chi_1-(c+\gamma)(\chi_0-\chi_1+\frac{p\theta R}{1-p}))}$

Note: Meaningful economic solutions require $0 \leq \beta, \phi \leq 1$. This restriction limits the range of admissible parameter values.

Table 2: The Baseline Case and Comparative Statics Results

	Baseline Case		$\boldsymbol{\chi_1}$= **0.075** ↑		$\boldsymbol{\gamma}$ = **0.0105** ↑		$\boldsymbol{\theta}$ = **0.7** ↓		$\boldsymbol{p}$ = **0.951** ↑	
	UC	C	UC	C	UC	C	UC	C	UC	C
β	0.2	0.251	0.25	0.226	0.262	0.268	NC	0.23	0.11	0.257
ϕ	0.63	0.753	0.81	0.752	0.76	0.752	NC	0.70	0.39	0.753

Note: UC = Unconstrained, C = Constrained, NC = No Change.