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Transactions Costs and the Equity Premium Puzzle

Sanghyun Hong

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Department of Economics and Finance
School of Business
University of Canterbury
Private Bag 4800, Christchurch
New Zealand

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Sanghyun Hong[1†]

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Abstract: Campbell and Cochrane's (1999b) habit formation model is able to resolve the equity premium and riskless interest rate puzzles, but only for high values of relative risk aversion. In this paper, I incorporate transactions costs in the Campbell and Cochrane model and find that the required level of relative risk aversion at the steady state reduces from 35 to 15. Thus, transactions costs seem able to reduce, but not completely solve, the remaining puzzle.

Keywords: Transaction Costs; Equity Premium Puzzle

JEL Classifications: G00; G12

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[1] Department of Economics and Finance, University of Canterbury, Christchurch, NEW ZEALAND

†Corresponding author: Sanghyun Hong. Email: sanghyun.hong@pg.canterbury.ac.nz.

Transactions Costs and the Equity Premium Puzzle

1 Introduction

In a seminal paper, Campbell and Cochrane (1999b) show that incorporating habit formation in a standard asset pricing model can successfully replicate many observed features of stock returns and thus largely resolve the equity premium puzzle of Mehra and Prescott (1985). However, their steady state solution requires relative risk aversion equal to 35, which seems implausibly high. As Cochrane (2016) points out:

> *Our model really does not solve the equity premium puzzle. The equity premium puzzle as now distilled includes the equity premium, the market Sharpe ratio, a low and stable risk-free rate, realistic consumption growth volatility, with a positive discount factor δ and low risk aversion. We have everything but low risk aversion. So far no model has achieved a full solution of the equity premium puzzle as stated.* (Cochrane, 2016, Page 7)

In this paper, I reexamine the equity premium puzzle by incorporating transactions costs into the Campbell and Cochrane's habit formation model. The objective is to determine whether the incorporation of such costs can sufficiently reduce the required level of relative risk aversion while at the same time retaining the main properties of Campbell and Cochrane (e.g., high equity premium, low riskless rate, and the market Sharpe ratio).

Transactions costs associated with holding a well diversified portfolio have been high during the last century (McGrattan and Prescott, 2001, 2003; Jones, 2002), so they should not be ignored. The particular costs I focus on are fund diversification costs and dividend income taxes. In reality, investor pays taxes not only on dividend income, but also on capital gains. In order to examine the effects of capital gain taxes on equity prices in a model, I need to estimate marginal tax rate that applies to a marginal investor. However, estimating the marginal capital gain tax rate is a challenging task due to data unavailability (McGrattan and Prescott, 2003). Therefore, I focus on dividend income taxes and consider it as conservative (lower bound) estimate for tax costs.

This study consists of three steps. I first estimate average fund diversification costs and average marginal dividend income tax rates over the 1947–1995 period.[1] Then, I incorporate these costs into the budget constraint of Campbell and Cochrane's habit model and solve for equity prices (e.g., price consumption ratio). Lastly, I calibrate the utility curvature parameter to match the observed historical Sharpe ratio of 0.43 and examine whether incorporation of the fund diversification costs and dividend income taxes can solve the puzzle.

I find that the utility curvature parameter of 0.36 successfully replicates the key features, such as the equity premium, low riskless interest rate, and the market Sharpe ratio. More importantly, in this model, when the utility curvature parameter is 0.36, the steady state relative risk aversion becomes 15 which is much smaller than what is implied in the original work of Campbell and Cochrane.

The paper closest to mine is McGrattan and Prescott (2003). They argue that the difference between taxes and costs adjusted equity returns (less than 5%) and the average returns on US long-term debt assets (almost 4%) is less than 1%, and therefore, there is no equity premium puzzle. However, their study and mine are different in several ways. Rather than focusing on the difference between the observed average market equity returns and transactions costs, I numerically solve Campbell and Cochrane's habit formation model with transactions costs and characterize model predictions of equity prices. Instead of using long-term bonds, I use 90-day US treasury bill rate as riskless interest rate, which is consistent with Mehra and Prescott and Campbell and Cochrane. Furthermore, I calibrate a model parameter not only to explain the large equity premium, but also to explain high stock market returns volatilities.

In the next section, I outline the transactions costs incorporated habit model. Then, Section 3 and Section 4 present equity price estimation and simulation results respectively, and Section 5 discusses investor relative risk aversion. Finally, Section 6 concludes the

[1] I obtain the datasets from Professor McGrattan's webpage, `http://users.econ.umn.edu/~erm/research.php`.

paper.

2 The Model

Following Campbell and Cochrane (1999b), I assume that investors have a power utility function, where utility is derived from the difference between consumption and a slowly adjusting consumption habit. That is:

$$u(C_t) = \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}$$

where C_t is consumption at time t, X_t is consumption threshold (habit consumption), and $\gamma > 0$ is the relative risk aversion coefficient. Infinitely-lived investors maximize their lifetime utility by choosing optimal level of consumption and investment each period.

$$\begin{aligned} \underset{C_t,\, Z_{t+1}}{\text{maximize}} \quad & E\left[\sum_{t=0}^{t=\infty} \delta^t u(C_t)\right] \\ \text{subject to} \quad & C_t + Z_{t+1}P_t \leq Z_t P_t(1-f_c) + Z_t Y_t(1-\tau_d) \end{aligned} \tag{1}$$

where $\delta > 0$ is the subjective time discount factor and $\gamma > 0$ is the utility curvature parameter. Z_t and Y_t in the budget constraint are household risky asset holdings and dividend income respectively. C_t is consumption at time t and P_t is the price of risky asset at time t. Investors in this economy are required to pay f_c fraction of fees relative to the value of risky assets when rebalancing the risky asset holdings at the end of each period t. In addition to the fund diversification cost, f_c, investors are also required to pay dividend income tax, τ_d.

Let

$$S_t \equiv \frac{C_t - X_t}{C_t} \tag{2}$$

denote the consumption surplus ratio, which is an indicator of the economy's state. For example, $S_t = 0.02$ implies that the consumption at time t is 2% above the consumption threshold, X_t. Therefore, low S_t represents a 'hungrier' state and high S_t denotes a 'good' economic state.

The processes of log consumption surplus ratio (e.g., $s_t = \ln(S_t)$) and log normally distributed consumption growth are defined as:

$$\ln(S_{t+1}) = (1-\phi)\ln\left(\bar{S}\right) + \phi\ln(S_t) + \lambda(\ln(S_t))\,\nu_{t+1} \tag{3}$$

$$\ln\left(\frac{C_{t+1}}{C_t}\right) \approx \Delta c_{t+1} = \bar{g} + \nu_{t+1} \qquad \text{where } \; \nu_{t+1} \sim \text{NIID}(0, \sigma_c^2) \tag{4}$$

where $\bar{S}$ is the steady state consumption surplus ratio (unconditional mean of S_t), ϕ is consumption persistency coefficient, Δc_{t+1} is $\ln C_{t+1}$ - $\ln C_t$, and $\lambda(\ln(S_t))$ is investors' sensitivity to consumption shocks. As in equation (4), the log consumption grows at a constant rate, $\bar{g}$, with homoskedastic innovations, ν_{t+1}. Plase note that all logs in this paper are natural logs.

Equation (5) shows the local coefficient of relative risk aversion (η_t) of this economy.

$$\eta_t = -\frac{C_t u_{CC}(C_t)}{u_C(C_t)} = \frac{\gamma}{S_t} \tag{5}$$

It indicates that local relative risk aversion is the inverse of S_t. Investor becomes very risk-averse when consumption is close to the habit level.

$$\lambda(\ln(S_t)) = \begin{cases} \frac{1}{\bar{S}}\sqrt{1 - 2\left(\ln(S_t) - \ln(\bar{S})\right)} - 1 & \text{if} \quad S_t \leq S_{max} \\ 0 & \text{if} \quad S_t \geq S_{max} \end{cases} \tag{6}$$

$$\bar{S} = \sqrt{\frac{\gamma\sigma^2}{1-\phi}} \tag{7}$$

$$\ln(S_{max}) = \ln(\bar{S}) + \frac{1}{2}(1 - \bar{S}^2) \tag{8}$$

Lastly, the equations (6), (7) and (8) present the reverse engineered consumption shock sensitivity function, $\lambda(\ln(S_t))$, steady state consumption surplus ratio, $\bar{S}$, and log maximum consumption surplus ratios, $\ln(S_{Max})$, respectively. The sensitivity function decreases with S_t. It implies that investors become more and more anxious about consumption shocks as the consumption level gets close to the habit level, and therefore, this is what creates a counter-cyclical precautionary savings demand.

The pricing equation in this economy is given by

$$1 = E\left[M_{t+1}\frac{P_{t+1}(1-f_c)+Y_{t+1}(1-\tau_d)}{P_t}\right] = E\left[M_{t+1}R_{t+1}\right] \tag{9}$$
$$\text{where} \quad M_{t+1} = \delta\frac{u_C(C_{t+1})}{u_C(C_t)} = \delta\left(\frac{S_{t+1}}{S_t}\right)^{-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}.$$

R_{t+1}, in equation (9), is after cost (fund diversification costs and dividend income taxes) returns and M_{t+1} is stochastic discount factor. Equation (10) shows the log riskless interest rate in this economy. Counter-cyclical variations in precautionary savings demand cancel out intertemporal substitution effect, making the riskless interest constant.

$$r_{f,t} = -\ln(\delta) + \gamma\bar{g} - \left(\frac{\gamma}{\bar{S}}\right)^2\frac{\sigma_c^2}{2} \quad \text{for} \quad S_t \leq S_{Max} \tag{10}$$

3 Price-Consumption Ratio and Expected Returns

In this section, I consider stocks as a claim to the consumption stream. Because consumption surplus ratio is the only state variable in this economy, the price-consumption ratio should be a function of $s_t = \ln(S_t)$. The price consumption ratio, P_t/C_t, should satisfy the condition

$$\frac{P_t}{C_t}(s_t) = E\left[M_{t+1}\frac{C_{t+1}}{C_t}\left((1-f_c)\frac{P_{t+1}}{C_{t+1}}(s_{t+1}) + 1 - \tau_d\right)\right]. \tag{11}$$

The price-consumption ratios in equation (11) does not have a closed-form solution, so I numerically solve them using a fixed-point method over a grid of values for s_t.[2] Given the price-consumption ratio calculated from equation (11), I compute expected before and after costs returns for each state s_t.

The conditional expected after cost returns, $E_t[R_{t+1}]$, is the difference between the

[2] I use 301 grid points, which is much finer than the 17 grid points used in Campbell and Cochrane (1999b). The 301 grid points consists of 300 equally spaced intervals over $[0.001, S_{Max}]$ plus $\bar{S}$. Different set of grids yield noticeably different price-consumption ratios, but it has no effect on riskless interest rates, and has only a marginal effect on Sharpe ratio. See Wachter (2005) for detail.

expected before cost returns, $E_t[\tilde{R}_{t+1}]$, and the expected relative costs, $E_t[L_{t+1}]$. All three terms can also be written as a function of the consumption surplus ratio:

$$
\begin{aligned}
E\left[R_{t+1}\left(s_{t+1}\right)|s_t\right] &= E\left[\left.\frac{P_{t+1}\left(1-f_c\right)+C_{t+1}\left(1-\tau_d\right)}{P_t}\right|s_t\right] \\
&= E\left[\tilde{R}_{t+1}\left(s_{t+1}\right)-L_{t+1}\left(s_{t+1}\right)|s_t\right]
\end{aligned} \tag{12}
$$

where

$$
\begin{aligned}
R_{t+1}\left(s_{t+1}|s_t\right) &= \frac{C_{t+1}}{C_t}\frac{\left(\left(1-f_c\right)\frac{P_{t+1}}{C_{t+1}}\left(s_{t+1}\right)+\left(1-\tau_d\right)\right)}{\frac{P_t}{C_t}\left(s_t\right)} \\
\tilde{R}_{t+1}\left(s_{t+1}|s_t\right) &= \frac{P_{t+1}+C_{t+1}}{P_t} = \frac{C_{t+1}}{C_t}\frac{\left(1+\frac{P_{t+1}}{C_{t+1}}\left(s_{t+1}\right)\right)}{\frac{P_t}{C_t}\left(s_t\right)} \\
L_{t+1}\left(s_{t+1}|s_t\right) &= \frac{P_{t+1}f_c+C_{t+1}\tau_d}{P_t} = f_c\frac{C_{t+1}}{C_t}\frac{\frac{P_{t+1}}{C_{t+1}}\left(s_{t+1}\right)}{\frac{P_t}{C_t}\left(s_t\right)}+\tau_d\frac{C_{t+1}/C_t}{\frac{P_t}{C_t}\left(s_t\right)}
\end{aligned}
$$

Once the price-consumption ratios have been computed from equation (11), the estimates for conditional expected before and after costs returns as well as expected relative costs in equation (12) are straightforward.

The expected relative costs have two parts. The first component is f_cP_{t+1}/P_t, measured by the fund diversification costs (f_cP_{t+1}) relative to P_t. The latter component is τ_dC_{t+1}/P_t, measured by dividend income taxes (τ_dC_{t+1}) relative to P_t. Given the independent and identically distributed consumption growth, the expected relative dividend income tax costs can be expressed as $E[L_{t+1}(s_{t+1})|s_t] = (\tau_d/(P_t/C_t))E[C_{t+1}/C_t] = (\tau_de^{\bar{g}+0.5\sigma_c^2})/(P_t/C_t)$. It implies that if expected stock returns are counter-cyclical (e.g., low equity price during recessions and vice versa), the expected relative dividend income tax costs would also be counter-cyclical. The conditional expected relative fund diversification costs at the steady state is $f_ce^{\bar{g}+0.5\sigma_c^2}$. However, for $s_t \neq \bar{s}$, the conditional expected relative fund diversification costs cannot be further simplified.

In order to numerically solve the price-consumption ratios in equation (11), it is necessary to choose parameter values. Table 1 presents four sets of model parameters. The first four values are Campbell and Cochrane's consumption and real log riskless

Table 1: Parameter Choices

Parameter	Variable	Value
Consumption and Riskless Interest Rate Variables:		
Mean consumption growth (%)*	$\bar{g}$	1.89
Standard deviation of consumption growth (%)*	σ_c	1.50
Consumption Persistency coefficient*	ϕ	0.87
Log riskless interest rate (%)*	r_f	0.94
Fund Costs and Dividend Tax (1947-1995 period):		
Fund Diversification Cost (%)*	f_c	2.16
Marginal Dividend Income Tax Rates (%)	τ_d	38.23
Calibrated Utility Curvature Parameter:		
Utility curvature	γ	0.36
Model Implied Variables:		
Subjective discount factor*	δ	0.97
Steady-state consumption surplus ratio	$\bar{S}$	0.024
Maximum consumption surplus ratio	S_{max}	0.040
Steady State Relative Risk Aversion	$\gamma/\bar{S}$	14.88

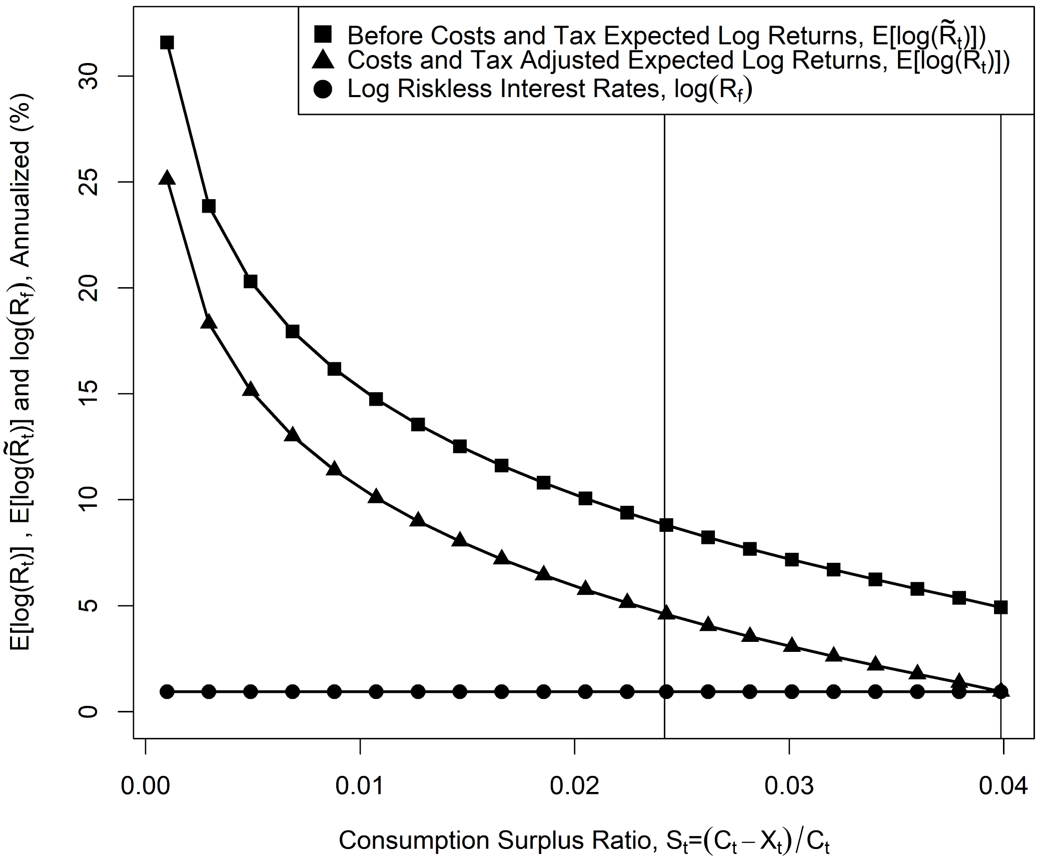
* parameter values are annualized, e.g., $12\bar{g}$, $\sqrt{12}\sigma_c$, $12r^f$, ϕ^{12}, δ^{12} and $12f_c$.

interest rate estimates. They estimate these parameter values from 1946–2017 US annual data. The next two rows presents the average annualized fund diversification costs and the average marginal dividend income taxes estimated from the same time periods. I estimate the corresponding quantities from Professor McGrattan's dataset. The next row shows the calibrated utility curvature parameter, γ. I calibrate $\gamma = 0.36$ to match the historical market Sharpe ratio of 0.43. The last four rows present the model implied parameter values.

Given the parameter values in Table 1, I first compute the price-consumption ratios in equation (11). I then calculate the before and after costs expected log returns, $E[\ln(\tilde{R}_{t+1})]$ and $E[\ln(R_{t+1})]$ respectively. I also estimate expected log relative costs, $E[\ln(L_{t+1})]$, as well as log riskless interest rate, $r^f = \ln(R_f)$ for each state, s_t.

Figure 1 presents the expected before and after costs consumption claim log returns and the riskless interest rates. When consumption gets close to the habit level ($S_t \to 0$), conditional expected returns rise dramatically (and hence risk premium). Not surprisingly, before costs expected log returns are always greater than after costs expected log returns. The difference between the two would provide close approximation for L_{t+1} in equation (12). Perhaps it is not very clear from Figure 1, the difference between the

Figure 1: Expected before/after Fund Costs and Tax Log Returns and Log Riskless Rate
This figure presents the annualized expected fund costs and tax adjusted log returns, $E[\ln(\tilde{R}_{t+1})]$ and expected cost unadjusted log returns, $E[\ln(R_{t+1})]$. It also presents constant log riskless interest rates, $\ln(R_f)$.



two lines monotonically decreases as S_t rises. For example, expected L_{t+1} decreases from 6.46% for $S_t = 0.001$ to 3.98% for $S_t = S_{Max}$. In other words, an investor requires more return for bearing the costs during recessions than during booms.

4 Simulation

To examine the model predictions for asset returns with the costs, I simulate 1,000,000 months of artificial data and calculate descriptive statistics for various variables. Table 2 shows the implications of the model for equity returns with and without the costs. The first column presents simulation results reported in Campbell and Cochrane. The second column presents my simulation results of the habit model with the costs. The last column

Table 2: Simulation Results

This table presents various statistics calculated from 1,000,000 simulated artificial data. The model is simulated at a monthly frequency and the statistics are converted into annualized values. The first column presents simulation results reported in Campbell and Cochrane (1999b). The second column presents simulation results when both the fund costs and taxes are incorporated into the model. The last column presents the corresponding (annualized) descriptive statistics estimated from the U.S. historical data for 1947–1999 period. Note that $\tilde{r}_M = \ln(\tilde{R}_M)$, $r_M = \ln(R_M)$, and $r_f = \ln(R_f)$.

Variable	Campbell and Cochrane	My Results	Data
Parameter Values:			
f_c (%)	0	2.16	2.16
τ_d (%)	0	38.23	38.23
γ	2	0.36	
$\gamma/\bar{S}$	35.08	14.88	
Simulation Results:			
$E[r_f]$ (%)	0.94	0.92	0.94
$E[\tilde{r}_M - r_f]$ (%)	6.64	6.73	6.69
$\sigma(\tilde{r}_M - r_f)$ (%)	15.2	15.58	15.7
$E[\tilde{r}_M - r_f]/\sigma(\tilde{r}_M - r_f)$	0.43	0.43*	0.43
$E[\tilde{R}_M - R_f]/\sigma(\tilde{R}_M - R_f)$	0.50	0.50	0.50
$exp\left(E\left[log\left(\frac{P}{C}\right)\right]\right)$	18.3	18.5	24.7
$\sigma\left(log\left(\frac{P}{C}\right)\right)$	0.27	0.28	0.26

* I calibrate the utility curvature parameter, γ, to replicate the Sharpe ratio of 0.43.

shows the corresponding statistics estimated from US historical data over the 1947–1995 period. All the descriptive statistics in Table 2 are annualized.

In the first column, with the calibrated utility curvature parameter $\gamma = 2$, Campbell and Cochrane successfully explains (1) the large equity premium, (2) low riskless interest rate and (3) the market Sharpe ratio, but their solution requires high relative risk aversion. The required steady state relative risk aversion is 35.

The second column presents my simulation results of the model with the costs. I calibrate the risk aversion parameter $\gamma = 0.36$ to match the historical market Sharpe ratio of 0.43. I find that the model with costs can also successfully explain (1) the equity premium, (2) low riskless interest rate and (3) and the market Sharpe ratio, but with much smaller steady state relative risk aversion of 14.88.[3]

[3]It is noteworthy that when γ becomes smaller, the average simulated riskless interest rates goes slightly below the target riskless interest rate of 0.94%. See Appendix for detail.

5 Relative Risk Aversion

Campbell and Cochrane show that the local coefficient of relative risk aversion (η_t) in equation (5) and wealth based relative risk aversion (rra_t) are not the same in this economy. When relative risk aversion is measured as investor's attitudes toward pure wealth bets, rra_t would depend on the curvature of investor's value function. The value function, $V(W_{i,t}, W_{a,t}, S_{a,t})$, depends on investor i's wealth, $W_{i,t}$, and on other aggregate economic variables, $W_{a,t}$ and $S_{a,t}$. By applying the Envelope condition $u_C = V_W$, rra_t can be written as a function of the local coefficient of relative risk aversion:

$$rra_t = -\frac{WV_{WW}}{V_W} = \eta_t \frac{\partial \ln C_t}{\partial \ln W_t} \tag{13}$$

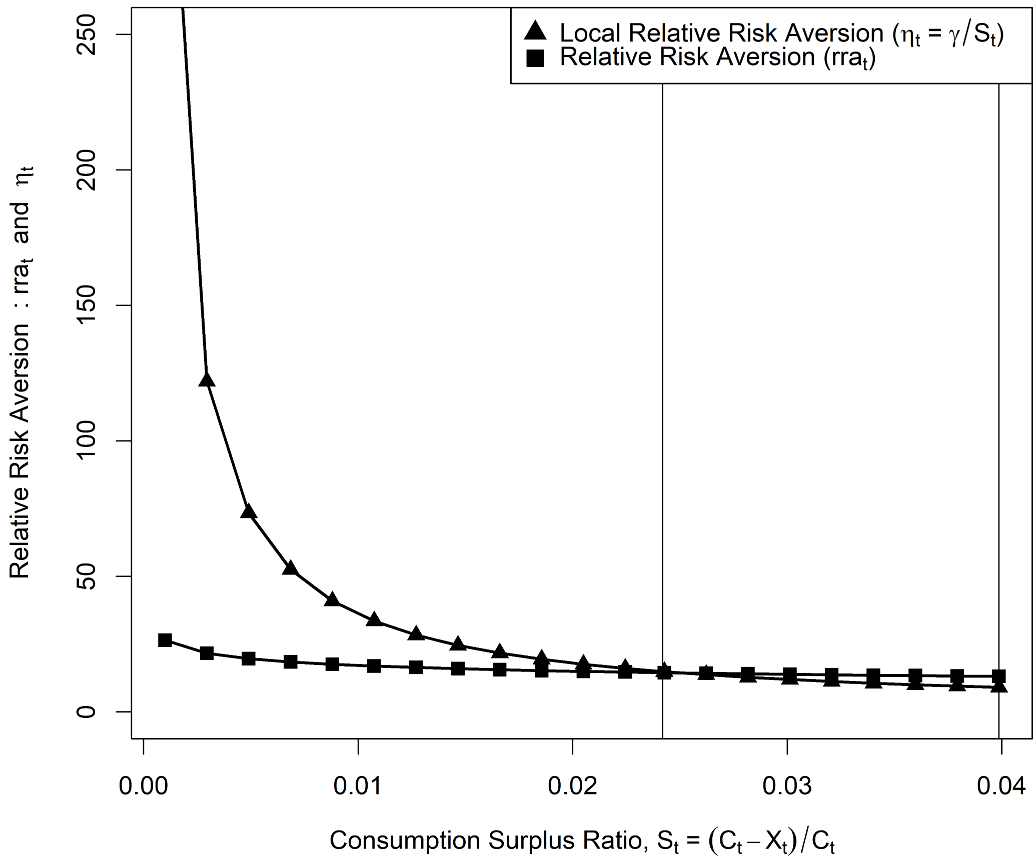
As equation (13) suggests, when rra_t is defined from the value function, rra_t and η_t are not the same in this economy. $rra_t > \eta_t$ would occur if consumption rises more than proportionally to an increase in individual wealth (e.g., $\frac{\partial \ln C}{\partial \ln W} > 1$). In Campbell and Cochrane where $\gamma = 2$, they find $\frac{\partial \ln C}{\partial \ln W} > 1$, and therefore $rra_t > \eta_t$ for every economic state, S_t.

Intuitively, an increase in investor's wealth leads to an increase in consumption over habit, and this increase will reduce investor's demand for precautionary savings, making a further increase in consumption. Therefore, consumption can rise more than proportionally to an increase in individual wealth. Numerically, Campbell and Cochrane find that at the steady state, rra_t is about twice greater than η_t. Furthermore, rra_t varies dramatically over economic business cycles: rra_t increases to several hundreds as C_t approaches to the habit level, X_t.[4]

Interestingly, the low $\gamma = 0.36$ implied by the steady-state solution with transactions costs generates (1) very stable rra_t over economic business cycles and (2) $rra_t < \eta_t$ for low S_t. Figure 2 illustrates the behaviours of the local relative risk aversion (η_t) and relative risk aversion (rra_t) for Campbell and Cochrane's habit formation model with transactions costs.

[4]See Section 4 of Campbell and Cochrane (1999a) for detail.

Figure 2: Relative Risk Aversion (rra_t) and Local Relative Risk Aversion (η_t)



First of all, compared to rra_t in Campbell and Cochrane's economy without transactions costs (where $\gamma = 2$), rra_t in Figure 2 is much low and stable. Both η_t and rra_t are about 15 at the steady state. Stability of rra_t is largely related to a smaller γ. Equation (13) suggests that var(rra_t) is directly proportional to γ^2. Reducing γ from 2 to 0.36 lowers var(rra_t) by a factor of about 30. Therefore, smaller γ due to the incorporation of transactions costs not only provides lower level of relative risk aversion at the steady state, but also provides much more stable investor relative risk aversion. In other words, the model with transactions costs can explain countercyclical stock market returns without having very large fluctuations in rra_t.

Secondly, I find that $rra_t < \eta_t$ for low S_t. This puzzling finding can also be explained by low γ. Low S_t means that consumption, C_t, is chosen to be close to the habit level, X_t. With low γ (e.g., $\gamma = 0.36$), the investor would not have very strong desire to smooth consumption, and he would still find investment more attractive than consumption at low

S_t. So, if the investor gets more wealth, he would not spend all of it on extra consumption (e.g., $\frac{\partial \ln C}{\partial \ln W} < 1$). By contrast, the more risk averse investor ($\gamma = 2$) would have much stronger desire for consumption smoothing and would be a lot less happy about low S_t. So, any extra wealth he gets would be devoted entirely to consumption. This explains the different behaviours of rra_t in Campbell and Cochrane and the economy with transactions costs.

6 Conclusion

I find the incorporation of fund diversification costs and dividend income taxes into Campbell and Cochrane's habit model can not only successfully explain (1) the equity premium, (2) low riskless interest rate and (3) and the market Sharpe ratio, but also require much lower and stable relative risk aversion than what is implied in the original work of Campbell and Cochrane.

Then, is the equity premium puzzle solved? Unfortunately, no. I find that incorporating the observed level of fund diversification costs and dividend income taxes can reduce the required level of relative risk aversion from 35 to 15 (about a 58% reduction). However, the steady state relative risk aversion of 15 seems still quite high. Hence, I conclude that the incorporation of costs and taxes can reduce the equity premium puzzle substantially, but it does not completely solve the puzzle.

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Appendix. Fragile Riskless Interest Rates

One of the key achievements in Campbell and Cochrane (1999b) is that their model can match the low and constant riskless interest rates. The equation (A.1) shows the riskless interest rate in their model. The riskless interest rate is constant when $S_t < S_{Max}$, but it starts to decrease as S_t goes beyond S_{Max}. When S_t is above S_{Max}, the intertemporal substitution effects dominate the precautionary savings demand, making riskless rate decreasing.

$$r_{f,t+1} = \begin{cases} -\ln(\delta) + \gamma\bar{g} - \left(\frac{\gamma}{\bar{S}}\right)^2 \frac{\sigma_c^2}{2} & \text{if} \quad S_t \leq S_{Max} \\ -\ln(\delta) + \gamma\bar{g} - \gamma(1-\phi)(\ln(S_t) - \ln(\bar{S})) - \frac{\gamma^2\sigma_c^2}{2} & \text{if} \quad S_t \geq S_{Max} \end{cases} \tag{A.1}$$

Is $S_t > S_{Max}$ possible? Very unlikely but possible. Because log consumption growth is normally distributed without bounds, a realization of a very large consumption growth shock, ν_{t+1}, can push S_t above S_{Max}. Equation (A.2) shows the probability of $S_t > S_{Max}$ at any given S_{t-1}.

$$Pr(S_t \geq S_{Max}|S_{t-1}) = Pr\left(\nu_t \geq \frac{1}{\lambda(\ln(S_{t-1}))}\left[\frac{1}{2}\left(1-\bar{S}^2\right) - \phi\big(\ln(S_{t-1}) - \ln(\bar{S})\big)\right]\middle| S_{t-1}\right) \tag{A.2}$$

Note that the consumption growth shock is normally distributed with zero mean and homoskedastic variance (e.g., $\nu_t \sim NIID(0, \sigma_C^2)$) and that both the sensitivity function, $\lambda(\ln(S_t))$, and $\bar{S}$ are a function of γ. Therefore, the probability in equation (A.2) is also a function of γ. The probability of having $S_t > S_{Max}$ (and hence $r_{f,t+1}$ <0.94%) will rise as the utility curvature parameter, γ, decreases.

To see the relationship between the probability of $S_t > S_{Max}$ and γ closely, I evaluate the equation (A.2) at $S_{t-1} = \bar{S}$ (probability of economy reaching above S_{Max} today when the economy was at the steady state yesterday). The equation (A.3) suggests that if the economy receives consumption growth shock $\frac{1}{2}\left(\frac{\sigma_c\gamma}{1-\phi} + \sqrt{\frac{\gamma}{1-\phi}}\right)$ standard deviation above from the mean, the economy will eventually be at $S_t > S_{Max}$. This clearly shows that the likelihood of reaching $S_t > S_{Max}$ increases as γ decreases.

$$
\begin{aligned}
Pr(S_t \geq S_{max}|S_{t-1} = \bar{S}) &= Pr\left(\frac{\nu_t}{\sigma_c} \geq \frac{1}{2}\left(\frac{\sigma_c\gamma}{1-\phi} + \sqrt{\frac{\gamma}{1-\phi}}\right)\middle| S_{t-1} = \bar{S}\right) \\
&= Pr\left(z_t \geq \frac{1}{2}\left(\frac{\sigma_c\gamma}{1-\phi} + \sqrt{\frac{\gamma}{1-\phi}}\right)\middle| S_{t-1} = \bar{S}\right) \qquad \text{(A.3)} \\
&\text{where} \quad z_t \sim \text{NIID}(0,1)
\end{aligned}
$$

For the given parameter values in Table 1 and when $\gamma = 2$, $\frac{1}{2}\left(\frac{\sigma_c\gamma}{1-\phi} + \sqrt{\frac{\gamma}{1-\phi}}\right) \approx 6.96$. This implies that when the economy is at the steady state at time $t-1$, S_t will be above S_{Max}, if the consumption growth shock is drawn 6.96 standard deviation above from the mean. On the other hand, when $\gamma = 0.36$, $\frac{1}{2}\left(\frac{\sigma_c\gamma}{1-\phi} + \sqrt{\frac{\gamma}{1-\phi}}\right) \approx 2.86$. The probability of $Pr\left(z_{t+1} > 6.96\right)$ and $Pr\left(z_{t+1} > 2.86\right)$ are 1.72×10^{-12} and 2.11×10^{-3} respectively.