DEPARTMENT OF ECONOMICS AND FINANCE

SCHOOL OF BUSINESS AND ECONOMICS

UNIVERSITY OF CANTERBURY

CHRISTCHURCH, NEW ZEALAND

Systematic Liquidity Risk Premia

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WORKING PAPER

No. 15/2020

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August 2020

Abstract: This paper examines the β_4 liquidity risk premium documented in Acharya and Pedersen (2005). We decompose this premium into two components: the covariation of liquidity costs with (i) market dividend growth shocks and (ii) shocks to the variance of market returns. In 1963-2017 US stock market data, the former is approximately three times larger than the latter. Liquidity volatility is primarily incorporated in stock prices via its common variation with business, rather than financial, shocks.

Keywords: Liquidity Risk; Asset Pricing

JEL Classifications: G00, G12

Acknowledgements: For helpful comments on earlier versions of this paper, we are grateful to Alan Stent, Bill Rea, Ben Marshall, Nuttawat Visaltanachoti, Xing Han, Mudassar Hasan, and seminar participants at Canterbury, the 2018 and 2019 NZ Finance Colloquium, and the 2018 NZ Finance Meeting. As always, none bear any responsibility for remaining errors and ambiguities.

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1 Introduction

Several studies show that illiquidity affects asset prices not only as a direct cost (Amihud and Mendelson, 1986; Brennan and Subrahmanyam, 1996; Jones, 2002) but also as a systematic risk factor (Pastor and Stambaugh, 2003; Acharya and Pedersen, 2005; Amihud, 2014). In an important contribution, Acharya and Pedersen develop a liquidity-adjusted capital asset pricing model (henceforth LCAPM) in which systematic liquidity risks affect expected returns. In the LCAPM, the overall beta equals the sum of the the standard beta and three liquidity betas: (i) β_2 – the covariation between firm-specific liquidity costs and market liquidity costs, (ii) β_3 – the (negative) covariation between firm stock returns and market liquidity costs, and (iii) β_4 – the (negative) covariation between firm liquidity costs and market returns. The intuition for these additional betas is that volatility in future liquidity costs introduces an additional element of risk to future net stock returns which, if correlated with net market returns, adds to a stock's systematic risk and hence to its risk premium. Using 1964-99 US stock market data, Acharya and Pedersen estimate that the expected return premium associated with (i)–(iii) is 0.16%, 0.08%, and 0.82% respectively, from which they conclude that β_4 is easily the most important source of liquidity risk.

As Acharya and Pedersen (2005) point out (p.398): "This liquidity risk (β_4) has not been studied before either theoretically or empirically." Somewhat surprisingly then, this result seems to have received little attention in the literature. In this paper, we examine more closely the nature and source of β_4 risk. In particular, we seek to identify, and quantify, the fundamental economic determinants of this risk. First, we use the Campbell (1991) return decomposition to show that β_4 can be written as the sum of two sub-betas: the covariation of liquidity costs with business (aggregate discounted dividend) shocks and the covariation of liquidity costs with financial (market risk premium) shocks.

Second, we estimate each of these sub-betas using 1964–2017 US stock market data. For portfolios sorted along a variety of dimensions, we find that the business shock beta is always larger than the financial shock beta, typically by a substantial multiple.

This result has a straightforward qualitative explanation. Adverse business and financial shocks both reduce the value of the market portfolio (and hence investor wealth), but risk premium shocks also improve future investment opportunities (Campbell and Vuolteenaho, 2004). Consequently, adverse business shocks induce a greater excess of sellers over buyers than do financial shocks, and hence have a greater impact on a stock's illiquidity. Our estimates quantify the magnitude of this fundamental intuition. As a result of this difference in betas, there is also a difference in associated risk premia. In particular, we find that the business shock liquidity risk premium for the most illiquid portfolo (0.68%) is almost three times as large as the corresponding financial shock premium (0.24%). Investor concerns about illiquidity apparently relate more to its covariation with adverse business shocks than with shocks to market risk premia. However, both liquidity risk premia are small for most portfolios, only becoming economically significant for the most illiquid, volatile, or small portfolios.

We emphasize that our focus is not on testing the LCAPM, that exercise having already been undertaken by Acharya and Pedersen (2005). Instead, our objective is to shed further light on the illiquidity risk they identify as being most important: its fundamental determinants, and the relative importance of these determinants.

In the next section, we outline the underlying theoretical relationships linking liquidity risk, expected returns, and business and financial shocks. Section 3 describes our data and the procedures we use to estimate betas and risk premia. Section 4 then presents the estimation results, while section 5 considers some extensions and robustness issues. Finally, Section 6 contains some concluding remarks.

2 The Determinants of Liquidity Risk

2.1 The liquidity-adjusted CAPM (LCAPM)

Acharya and Pedersen (2005) consider an overlapping generations world where investors choose investment portfolios to maximize expected utility over time. Because investors are required to pay transaction costs when selling risky assets, they care about net returns (i.e., return minus transaction costs) on these assets, but the sale of riskless assets is costless and short-selling of risky assets is not allowed. Under these conditions, Acharya and Pedersen show that:

$$E_{t-1}\left[R_{it}\right]-R_{ft}=E_{t-1}\left[C_{it}\right]+\frac{\mathrm{cov}_{t-1}(R_{it}-C_{it},R_{Mt}-C_{Mt})}{\mathrm{var}_{t-1}(R_{Mt}-C_{Mt})}\left(E_{t-1}\left[R_{Mt}-C_{Mt}\right]-R_{ft}\right) \quad (1)$$

where R_{it} is the time t return on company i stock, R_{ft} is the riskless interest rate paid at time t, R_{Mt} is the market portfolio return at time t, C_{it} is the time t cost of selling one share of firm i expressed as a proportion of purchase price, and C_{Mt} is the corresponding cost of liquidating the market portfolio. Under stationarity, Acharya and Pedersen show that equation (1) can be written more compactly as:

$$E\left[R_{it}\right]-R_{ft}=E\left[C_{it}\right]+\lambda\Bigg(\beta_{1i}+\beta_{2i}-\beta_{3i}-\beta_{4i}\Bigg) \quad (2)$$

where:

$$\lambda=E\left[R_{Mt}-C_{Mt}\right]-R_{ft}$$

$$\beta_{1i}=\frac{\mathrm{Cov}\left(R_{it},R_{Mt}-E_{t-1}\left[R_{Mt}\right]\right)}{\mathrm{Var}\left(R_{Mt}-E_{t-1}[R_{Mt}]-(C_{Mt}-E_{t-1}[C_{Mt}])\right)}$$

$$\beta_{2i}=\frac{\mathrm{Cov}\left(C_{it}-E_{t-1}\left[C_{it}\right],C_{Mt}-E_{t-1}\left[C_{Mt}\right]\right)}{\mathrm{Var}\left(R_{Mt}-E_{t-1}[R_{Mt}]-(C_{Mt}-E_{t-1}[C_{Mt}])\right)}$$

$$\beta_{3i}=\frac{\mathrm{Cov}\left(R_{it},C_{Mt}-E_{t-1}\left[C_{Mt}\right]\right)}{\mathrm{Var}\left(R_{Mt}-E_{t-1}[R_{Mt}]-(C_{Mt}-E_{t-1}[C_{Mt}])\right)}$$

$$\beta_{4i}=\frac{\mathrm{Cov}\left(C_{it}-E_{t-1}\left[C_{it}\right],R_{Mt}-E_{t-1}\left[R_{Mt}\right]\right)}{\mathrm{Var}\left(R_{Mt}-E_{t-1}[R_{Mt}]-(C_{Mt}-E_{t-1}[C_{Mt}])\right)}$$

In the absence of liquidity costs ($C_{Mt} = C_{it} = 0, \forall i$), equation (2) reduces to the standard CAPM. However, in the LCAPM, expected returns depend not only on the standard market beta (β_{1i}) but also on expected liquidity costs ($E[C_{it}]$) and three liquidity betas (β_{2i}, β_{3i}, and β_{4i}). β_{2i} is the liquidity commonality beta — the covariation between stock i's liquidity costs and market liquidity costs. It captures the notion that investors require greater compensation for holding a stock that becomes particularly illiquid during times when the market as a whole is illiquid.[1] β_{3i} is the covariation between stock i returns and market liquidity costs. Investors are prepared to pay a premium for stocks that provide a hedge against market illiquidity. Finally, β_{4i} is the covariation between stock liquidity costs and the market returns. Intuitively, investors require compensation for holding stocks that become illiquid during market downturns. Using US 1964-99 stock market data, Acharya and Pedersen estimate that the market price of the third of these liquidity risks is much greater than for the other two: the return premium associated with β_{4i} (0.82%) is about 5 and 10 times greater than that for β_{2i} and β_{3i} respectively.

Given its strong economic significance, β_{4i} risk clearly warrants further investigation. In the next section, we consider its underlying determinants.

2.2 The economic determinants of β_4

Campbell (1991) uses the Campbell and Shiller (1988) decomposition to show that stock return innovations satisfy:

$$r_t - E_{t-1}[r_t] \approx \Delta E_t \left[\sum_{j=0}^{\infty} \rho^j \Delta d_{t+j} \right] - \Delta E_t \left[\sum_{j=1}^{\infty} \rho^j r_{t+j} \right] \tag{3}$$

where ΔE_t denotes the change in expectations from $t-1$ to t, $r_t = ln(1+R_t)$ is the natural log of returns at time t, $\Delta d_t = ln(\frac{D_t}{D_{t-1}})$ is the natural log of time t growth in dividends D_t, and $\rho \in (0,1)$ is a constant approximately equal to the average ratio of the

[1] See, for example, Chordia et al., 2000; Hasbrouck and Seppi, 2001; Huberman and Halka, 2001; Coughenour and Saad, 2004.

stock price to the sum of the stock price and the dividend.

Equation (3) must also hold for the market portfolio with return r_M. Splitting the market portfolio return into a short-term interest rate (which we assume is $r_f \equiv ln(1 + R_f))$ and an excess return $\pi \equiv r_M - r_f$, and adding and subtracting $\Delta E_t \left[\sum_{j=1}^{\infty} \rho^j r_{ft+j}\right]$ from the right-side of the above equation, we obtain:

$$\begin{aligned} r_{Mt} - E_{t-1}\left[r_{Mt}\right] &\approx \Delta E_t \left[\sum_{j=0}^{\infty} \rho^j \Delta d_{t+j}\right] - \Delta E_t \left[\sum_{j=1}^{\infty} \rho^j r_{ft+j}\right] - \Delta E_t \left[\sum_{j=1}^{\infty} \rho^j \pi_{t+j}\right] \\ &= \eta_{dt} - \eta_{\pi t} \end{aligned} \tag{4}$$

where $\eta_{dt} = \Delta E_t \left[\sum_{j=0}^{\infty} \rho^j \Delta d_{t+j}\right] - \Delta E_t \left[\sum_{j=1}^{\infty} \rho^j r_{ft+j}\right]$ is the weighted present value of shocks to expectations about future dividend growth and $\eta_{\pi t} = \Delta E_t \left[\sum_{j=1}^{\infty} \rho^j \pi_{t+j}\right]$ is the weighted sum of future shocks to the market risk premium. If η_d is positive, then expectations about future dividend growth have risen relative to the returns available on the riskless asset, so investors substitute out of the latter into stocks, driving up stock prices now and hence the current return relative to expectation. If η_π is positive, then investors' expected compensation for bearing for risk has risen, driving down current stock prices and hence the current return relative to expectation. For ease of exposition, we henceforth refer to η_d as business shocks, and to η_π as financial shocks.

Assuming that log market return innovations $(r_{Mt} - E_{t-1}\left[r_{Mt}\right])$ closely approximate simple return innovations $(R_{Mt} - E_{t-1}[R_{Mt})$, then (4) can be substituted into the expression for β_4 to obtain:[2]

$$\beta_{4i} = \beta_{4i}^a + \beta_{4i}^b \tag{5a}$$

where:

$$\beta_{4i}^a = \frac{\text{Cov}\left(C_{it} - E_{t-1}\left[C_{it}\right], \eta_{dt}\right)}{\text{Var}\left(r_{Mt} - E_{t-1}[r_{Mt}] - (C_{Mt} - E_{t-1}[C_{Mt}])\right)} \tag{5b}$$

$$\beta_{4i}^b = \frac{\text{-Cov}\left(C_{it} - E_{t-1}\left[C_{it}\right], \eta_{\pi t}\right)}{\text{Var}\left(r_{Mt} - E_{t-1}[r_{Mt}] - (C_{Mt} - E_{t-1}[C_{Mt}])\right)} \tag{5c}$$

[2] In our data, simple and log market return innovations have a correlation coefficient exceeding 0.99.

Equations (5a)-(5c) reveal that β_4, the common variation between illiquidity (high C) and unexpected market returns, can be separated into two sub-betas. The first, β^a_{4i}, arises from common variation in illiquidity and business shocks; the second, β^b_{4i}, is due to common variation in liquidity (low C) and financial shocks. Intuitively, remembering that higher β_4 corresponds to lower risk, investors prefer stocks that provide high dividends when liquidity is low (high β^a_{4i}), or have high liquidity when subject to a risk premium shock (high β^b_{4i}).

The economic questions of interest surround the economic magnitudes of these two sub-betas. Are they of equal importance to investors? If not, how do they differ? What quantitative effect does each have on the cost of equity capital? The remainder of the paper investigates these issues.

3 Computation of Shocks to Illiquidity, Market Returns, and Economic Conditions

To estimate β^a_{4i} and β^b_{4i}, we first need to estimate time series for illiquidity innovations ($C_{it} - E_{t-1}[C_{it}]$ and $C_{Mt} - E_{t-1}[C_{Mt}]$), economic shocks (η_{dt} and $\eta_{\pi t}$), and market return innovations ($r_{Mt} - E_{t-1}[r_{Mt}]$). For this purpose, we use daily return and volume data in CRSP for all firms listed on NYSE and AMEX between 31 July 1962 and 31 December 2017. We subsequently form portfolios sorted on various criterion, with the first year's sort based on 1963 data; this leaves January 1964 – December 2017 data available for estimation. We exclude penny stocks (price less than \$5) and thinly traded stocks (15 or less days of return and volume data in any month) in order to minimize microstructure and idiosyncratic effects respectively. Our procedure consists of three steps:

1. Follow the approach used by Acharya and Pedersen (2005) to estimate the illiquidity innovations appearing in equations (5a)-(5c).

2. Use an ARMA model to estimate the market return innovations appearing in equations (5a)-(5c).

3. Use a fundamental asset pricing relationship together with the ARMA model in 2. to estimate the economic shocks appearing in equations (5a)-(5c).

3.1 Illiquidity innovations

To estimate illiquidity, we follow the procedure of Acharya and Pedersen (2005) and use:[3]

$$C_{it} = \min\left\{0.25 + 0.30 ILLIQ_{it} \cdot P_{Mt-1}, 30\right\}/100 \tag{6}$$

where P_{Mt-1} is the ratio of market portfolio capitalization at the end of $t-1$ to market portfolio capitalization at the end of July 1962, and

$$ILLIQ_{it} = \frac{1}{\mathrm{Days}_{it}} \sum_{d=1}^{\mathrm{Days}_{it}} \frac{|R_{itd}|}{V_{itd}}$$

is the Amihud (2002) month t measure of illiquidity. R_{itd} and V_{itd} are, respectively, the stock i return and trading volume (in \$ million) on day d in month t, so $ILLIQ_{it}$ measures, for each month, the average price sensitivity of stock i to a given volume of trading: greater sensitivity indicates higher illiquidity. Amihud and Noh (2018) document that, unlike volume based measures of illiquidity, $ILLIQ$ correctly identifies liquidity crises such as the October 1987 stock market crash and the financial crisis of 2007-2009. Moreover, in contrast to illiquidity measures based on bid-ask spreads, $ILLIQ$ measures the overall costs of selling stocks. The transformation in equation (6) converts $ILLIQ$ into a direct measure of trading costs and normalizes it for market movements.

As we explain below in section 4, we estimate β_{4a} and β_{4b} for portfolios rather than

[3] One potential problem here is that Acharya and Pedersen (2005) estimate the parameters 0.25 and 0.30 from their 1964-99 data set, but we do not have access to the spread data required to re-estimate these parameters for our extended 1963-2017 period. We therefore stick with the Acharya and Pedersen parameter values, but as a robustness check repeat our analysis using the 1964-99 time period and obtain very similar results to those reported below — see the appendix.

individual stocks. The measure of portfolio illiquidity C_{pt} corresponding to (6) is:

$$C_{pt} = \sum_{i \in p} w_{it} C_{it}$$

where w_{it} is the weight of stock i in portfolio p. To compute the illiquidity innovations $C_{pt} - E_{t-1}[C_{pt}]$ appearing in β_{4a} and β_{4b}, we estimate a modified AR(2) model:

$$C_{pt} = \alpha_0 + \alpha_1 C^*_{pt-1} + \alpha_2 C^*_{pt-2} + \mu_{pt} \tag{7}$$

where each C^*_{pt-s} is an adjustment of C_{pt-s} designed to neutralize the impact of changes in P_M (as opposed to illiquidity).[4] The illiquidity innovations are then given by the estimated residual series from (7). That is:

$$C_{pt} - E_{t-1}[C_{pt}] = \hat{\mu}_{pt} \tag{8}$$

and, for the market portfolio:

$$C_{Mt} - E_{t-1}[C_{Mt}] = \hat{\mu}_{Mt} \tag{9}$$

For the market portfolio, equation (7) has an R^2 of 0.68 when the market portfolio is equal-weighted and 0.83 when it is value-weighted.[5] Table 1 contains some additional summary statistics for market illiquidity. As expected, both the mean and volatility of illiquidity are higher for the equal-weighted portfolio than for the value-weighted portfolio. Regardless of weighting, the first-order autocorrelations are very close to zero.

[4]For more details, see Acharya and Pedersen (2005, section 4.3).

[5]For the equal-weighted market portfolio, we use an AR(3) version of (7) as this yields lower AIC and BIC values. As Acharya and Pedersen (2005) point out, equal weighting helps alleviate problems caused by the over-representation of large and liquid stocks in our sample.

Table 1: Summary Statistics for Market Illiquidity

Descriptive statistics summarising the characteristics of monthly market illiquidity costs. $E[C_M]$ is average illiquidity for the market portfolio and $\sigma[C_M]$ is the corresponding standard deviation. $\sigma(\Delta C_M)$ is the standard deviation of the market portfolio's illiquidity innovations as estimated from an AR(3) version of equation (7). ρ_M is the first-order autocorrelation for these innovations. The sample period is January 1964 – December 2017 (648 months).

	Market Portfolio	
	Equal-Weighted	Value-Weighted
$E[C_M]$	0.815	0.278
$\sigma[C_M]$	0.246	0.017
$\sigma(\Delta C_M)$	0.140	0.007
ρ_M	0.032	−0.018

3.2 Economic shocks and market return innovations

To compute η_{dt} and $\eta_{\pi t}$, we first assume that the expected market risk premium is constant, i.e., $E_t[\pi_{t+j}] = E_t[\pi_{t+1}], \forall j$. Then equation (4) can be written as:

$$\begin{aligned} r_{Mt} - E_{t-1}\left[r_{Mt}\right] &= \eta_{dt} - \left(\sum_{j=1}^{\infty} \rho^j E_t[\pi_{t+1}] - \sum_{j=1}^{\infty} \rho^j E_{t-1}\left[\pi_{t+1}\right] \right) \\ &= \eta_{dt} - \frac{\rho}{1-\rho} \left(E_t[\pi_{t+1}] - E_{t-1}[\pi_{t+1}] \right) \end{aligned} \tag{10}$$

Next, we make use of a result from asset pricing which states that the (log) market risk premium π is approximately proportional to the variance of the (log) market return:[6]

$$E_t[\pi_{t+1}] = E_t[r_{Mt+1}] - r_{ft+1} \approx \gamma \sigma_{t+1}^2 \tag{11}$$

[6]Equation (11) follows from the basic pricing equation (Cochrane, 2005, ch.1) assuming lognormal market returns and utility that is isoelastic in end-of-perod wealth, and ignoring a Jensen's Inequality term (which affects only the interpretation of ϕ in (12) and has no impact on our results). Similar equations can also be obtained in other ways, e.g., Merton (1980) and Huang and Litzenberger (1988).

where γ is a scalar approximating average investor risk aversion and $\sigma_t^2 = var_{t-1}(r_{Mt})$ is the variance of r_{Mt}. Substituting (11) into (10) yields:

$$r_{Mt} - E_{t-1}\left[r_{Mt}\right] = \eta_{dt} + \phi\left(\sigma_{t+1}^2 - E_{t-1}[\sigma_{t+1}^2]\right) \tag{12}$$

where $\phi = \frac{-\rho\gamma}{1-\rho}$.[7] Equation (12) is immediately recognizable as a regression equation of the form:

$$r_{Mt} - E_{t-1}\left[r_{Mt}\right] = \alpha_M + \phi\left(\sigma_{t+1}^2 - E_{t-1}\left[\sigma_{t+1}^2\right]\right) + \xi_{Mt} \tag{13}$$

which implies (hats denote estimations from data):

$$\hat{\eta}_{\pi t} = \hat{\phi}\left(\hat{\sigma}_{t+1}^2 - \hat{E}_{t-1}\left[\sigma_{t+1}^2\right]\right) \tag{14}$$

$$\hat{\eta}_{dt} = \hat{\alpha_M} + \hat{\xi}_{Mt} \tag{15}$$

That is, the predicted and residual values obtained from estimating equation (13) generate estimated time series for $\eta_{\pi t}$ and η_{dt}.

Estimation of (13) requires that we first compute unexpected market returns ($r_{Mt} - E_{t-1}\left[r_{Mt}\right]$) and market variance ($\sigma_{t+1}^2 - E_{t-1}\left[\sigma_{t+1}^2\right]$). For this, we apply ARMA(m, n) models to January 1964 – December 2017 data (648 months):

$$r_{Mt} = \alpha_M + \sum_{j=1}^{m} \psi_{jM} r_{Mt-j} + \sum_{k=1}^{n} \theta_{kM} \epsilon_{Mt-k} + \epsilon_{Mt} \tag{16}$$

$$\sigma_t^2 = \alpha_\sigma + \sum_{j=1}^{m} \psi_{j\sigma} \sigma_{t-j}^2 + \sum_{k=1}^{n} \theta_{k\sigma} \epsilon_{\sigma t-k} + \epsilon_{\sigma t} \tag{17}$$

where σ_{t-j}^2 is computed from daily returns in month $t - j$. Table A1 reports the results from estimating these models for various values of m and n. Although there is little between them, we adopt the ARMA (2,0) specification for market return innovations

[7] Consistent with French et al. (1987), the predicted relationship between expected return innovations and variance innovations is negative: an unexpected positive shock to variance raises expected returns and hence lowers current prices and returns.

Table 2: Computation of η_{dt} and $\eta_{\pi t}$

Panel A reports the results from estimating equation (13) – the regression of monthly market return innovations (equation (16)) on market variance innovations (equation (17)) – using January 1964–December 2017 monthly data (648 months). Terms in parentheses are Newey and West (1994) standard errors. Panel B contains some summary statistics for the estimates of η_{dt} and $\eta_{\pi t}$ obtained from the panel A regressions — see equations (14) and (15).

	Panel A: Regression			*Panel B: Summary Statistics*	
	(1)	(2)		$\eta_{d,t}$	$-\eta_{\pi t}$
α_M	0.000		Standard Deviation	0.055	0.023
	(0.002)		Minimum	-0.256	-0.343
ϕ	-5.499	-5.499	Maximum	0.220	0.144
	(0.795)	(0.796)	Autocorrelation	-0.087	0.007
Adj. R^2	0.144	0.144	Correlation with ϵ_{Mt}	0.924	0.381

and the ARMA (1,1) specification for market variance innovations, based on the Akaike information criterion (AIC), the Bayesian information criterion (BIC), and the first order autocorrelation (ACF(1)) statistic, which is close to zero for all specifications. Ljung-Box test statistics for 6- and 12-month lags are insignificant at conventional levels, indicating that the estimated return and variance innovations are time-independent.

We use the residuals from these specifications of (16) and (17) to compute $(r_{Mt} - E_{t-1}[r_{Mt}])$ and $(\sigma^2_{t+1} - E_{t-1}[\sigma^2_{t+1}])$ respectively. This allows us to estimate equation (13), with the results reported in panel A of Table 2. As our model implies that the constant is zero, we estimate specifications both with and without an intercept, but this turns out to make little difference: regardless of specification, the estimate of ϕ is negative (-5.463) and significant at the 0.01 level.

We apply these results to equations (14) and (15) to compute $\eta_{\pi t}$ and η_{dt} respectively, and report some summary statistics in Panel B of Table 2. Business shocks are almost twice as volatile as financial shocks, but both have low first order autocorrelations. The correlation between market return innovations and η_{dt} is 0.926 while the correlation between market return innovations and $-\eta_{\pi t}$ is 0.378, suggesting that unexpected changes in stock returns are more closely related to business shocks than to financial shocks.

4 Estimation of β_{4a}, β_{4b}, and risk premia

We now have the necessary information for computing the two sub-betas appearing in equations (5b) and (5c): illiquidity cost innovations can be obtained from equations (8) and (9), business and financial shocks from (14) and (15), and market return innovations from (16). Because the illiquidity measure C_{it} described in section 3.1 is likely to be a noisy quantifier of illiquidity for individual stocks, we eschew stock-level analysis and instead form portfolios in order to estimate the two sub-betas. The sorting is initially based on illiquidity (C_{pt}) and is updated annually, with year y illiquidity of every stock determined by year $y-1$ average ranking.[8] Portfolio illiquidity C_{pt} is computed on a value-weighted basis. As previously noted, we also form two market portfolios, one that is equal-weighted and one that is value-weighted.

4.1 Estimation of β_{4a} and β_{4b}

Table 3 reports estimates of β_{4a}, β_{4b}, and portfolio characteristics for a sample of the 25 illiquidity-sorted portfolios.[9] Average illiquidity costs $E[C_p]$ rise from 0.25% for the most liquid portfolio to 8.29% for the least liquid. This is associated with higher average excess returns $E[R_p]$ — rising from 0.47% per month for the most liquid portfolio to 0.95% for the least liquid, although the increase is not perfectly monotonic. Monthly turnover is roughly n-shaped, first rising with illiquidity, then falling. Finally, portfolio market capitalization increases monotonically with liquidity.

Regardless of whether the market portfolio is equal- or value-weighted, estimates of both β_{4ap} and β_{4bp} decline (i.e., become more negative) in an almost perfectly monotonic manner with illiquidity: portfolios with high illiquidity (high C_p) tend to also have greater sensitivity of liquidity to both business shocks and to financial shocks. More interesting are the differences between them. As illiquidity increases, β_{4ap} estimates increasingly

[8] Unsurprisingly, sorting on size produces very similar results, so we do not report these findings.

[9] The t-statistics are based on standard errors estimated using the Davison and Hinkley (1997, sect. 2.3) bootstrapping procedure for 10,000 simulations.

Table 3: Portfolio Betas and Characteristics for Illiquidity-Sorted Portfolios

This table reports betas and other characteristics of illiquidity-sorted portfolios during 1964-2017. All estimates are based on 648 monthly observations.***[CORRECT?]*** Illiquidity, as measured by C_p, is updated every January based on the previous year's illiquidity. Returns and illiquidity for portfolio p are computed on a value-weighted basis and on both an equal-weighted and value-weighted basis (see 'Weight' row) for the market portfolio. Terms in parentheses are t-statistics, based on bootstrap standard errors based computed from 10,000 simulated realizations. $E\left[C_p\right]$ column is average illiquidity of portfolio p and $\sigma(\Delta C_p)$ column is the standard deviation of portfolio p's illiquidity innovations. $E\left[R_{ep}\right]$ and $\sigma(R_{ep})$ are average and standard deviation of value weighted monthly portfolio excess returns for portfolio p. Turn is monthly ***[CORRECT?]*** portfolio turnover ***[HOW MEASURED?]*** and Size is equity market capitalization.

	Estimated Portfolio Betas				Portfolio Characteristics					
	β_{4ap} $(\cdot 100)$	β_{4bp} $(\cdot 100)$	β_{4ap} $(\cdot 100)$	β_{4bp} $(\cdot 100)$	$E\left[c_p\right]$ (%)	$\sigma\left(\Delta c_p\right)$ (%)	$E\left[r_{e,p}\right]$ (%)	$\sigma\left(r_p\right)$ (%)	trn (%)	Size (%)
Liquid	-0.00 (-2.50)	-0.00 (-1.48)	-0.00 (-3.01)	-0.00 (-1.21)	0.25	0.00	0.47	1.53	5.68	24.16
3	-0.02 (-6.27)	-0.00 (-3.10)	-0.02 (-6.29)	-0.00 (-2.98)	0.26	0.00	0.55	1.76	7.90	4.48
5	-0.04 (-6.86)	-0.01 (-3.30)	-0.05 (-7.05)	-0.00 (-2.91)	0.27	0.01	0.65	1.82	8.57	2.41
7	-0.06 (-6.63)	-0.01 (-3.01)	-0.07 (-6.53)	-0.01 (-2.94)	0.28	0.01	0.73	1.95	9.56	1.37
9	-0.10 (-7.11)	-0.02 (-3.66)	-0.12 (-7.36)	-0.01 (-3.29)	0.30	0.02	0.74	1.99	9.28	0.92
11	-0.19 (-7.17)	-0.03 (-3.00)	-0.23 (-7.25)	-0.02 (-2.84)	0.34	0.03	0.70	2.04	8.64	0.66
13	-0.27 (-6.17)	-0.04 (-3.22)	-0.36 (-6.37)	-0.04 (-2.74)	0.38	0.05	0.74	2.11	8.45	0.48
15	-0.50 (-6.83)	-0.05 (-3.12)	-0.59 (-6.64)	-0.05 (-2.82)	0.45	0.07	0.82	2.22	7.82	0.36
17	-0.68 (-6.25)	-0.08 (-2.81)	-0.80 (-6.17)	-0.07 (-2.58)	0.59	0.11	0.78	2.32	6.88	0.27
19	-1.08 (-6.48)	-0.14 (-3.07)	-1.25 (-6.19)	-0.14 (-2.94)	0.79	0.17	0.89	2.43	6.41	0.18
21	-1.91 (-7.38)	-0.13 (-1.43)	-2.39 (-7.25)	-0.11 (-1.13)	1.27	0.31	0.90	2.58	5.23	0.12
23	-2.68 (-6.15)	-0.39 (-1.75)	-3.07 (-4.86)	-0.46 (-1.73)	2.45	0.59	0.95	2.72	4.53	0.08
Illiquid	-5.56 (-5.02)	-2.01 (-2.38)	-8.06 (-5.32)	-2.09 (-1.93)	8.29	1.47	0.95	3.05	2.96	0.03
Weight	Equal	Equal	Value	Value						

diverge from β_{4bp} estimates. For the 5th most-liquid portfolio, β_{4ap} is four times β_{4bp}, but for the 21st most-liquid portfolio this multiple rises to 15. The β_{4ap} estimates are not only economically bigger, but also more precisely estimated and hence have higher t-statistics. Overall, the liquidity risk of illiquid portfolios is primarily due to such portfolios seeing liquidity dry up in the presence of adverse business shocks; by contrast, liquidity seems to be relatively unaffected by adverse financial shocks.

Such a difference is not, in a qualitative sense, surprising. Both adverse business shocks (which reduce the riskless present value of expected future dividend growth) and adverse financial shocks (which increase expected future market risk premia) lower investor wealth, but the latter also improve future investment opportunities and so should not result in the same loss of liquidity. Our analysis suggests this difference can be substantial.

4.2 Expected Return Premia

For portfolio $\ell = 1, ..., 25$, the incremental expected return premium associated with β_{4a} risk is calculated as:

$$-12 \cdot \lambda \cdot (\beta_{4a\ell} - \beta_{4a1}) \tag{18}$$

where β_{4a1} is the value of β_{4a} for the most liquid portfolio ($\ell = 1$) and λ is the monthly net market risk premium defined in equation (2). Similarly, the incremental expected return premium associated with β_{4b} risk is calculated as:

$$-12 \cdot \lambda \cdot (\beta_{4b\ell} - \beta_{4b1}) \tag{19}$$

Acharya and Pedersen (2005) use cross-sectional regressions based on (2) to estimate λ. That is, for each month t,they estimate regressions of the form:

$$r_{pt} - r_{ft} = \alpha + \kappa \hat{C}_p + \lambda \hat{\beta}_p + \epsilon_{pt} \tag{20}$$

where $\hat{C}_p$ is the pre-estimated average of C_{pt} and $\hat{\beta}_p \equiv (\hat{\beta}_{1p} + \hat{\beta}_{2p} - \hat{\beta}_{3p} - \hat{\beta}_{4ap} - \hat{\beta}_{4bp})$ is the pre-estimated net portfolio beta. Using their preferred specification, they estimate λ to be 1.512. In our longer time series, based on a weighted average from eight specifications of (20) (see Table A3 for details), we obtain $\lambda = 1.010$ (equal-weighted market portfolio) or 0.964 (value-weighted market portfolio).[10] For the most illiquid portfolio, the implied expected return premia from (18) and (19) are:

[10]For completeness, our estimates of β_{1p}, β_{2p}, and β_{3p} appear in Table A2.

$$\beta_{4a} \text{ risk premium} = -12 \cdot 1.010 \cdot (-0.056 - 0) = 0.68\% \text{ (equal-weighted)}$$

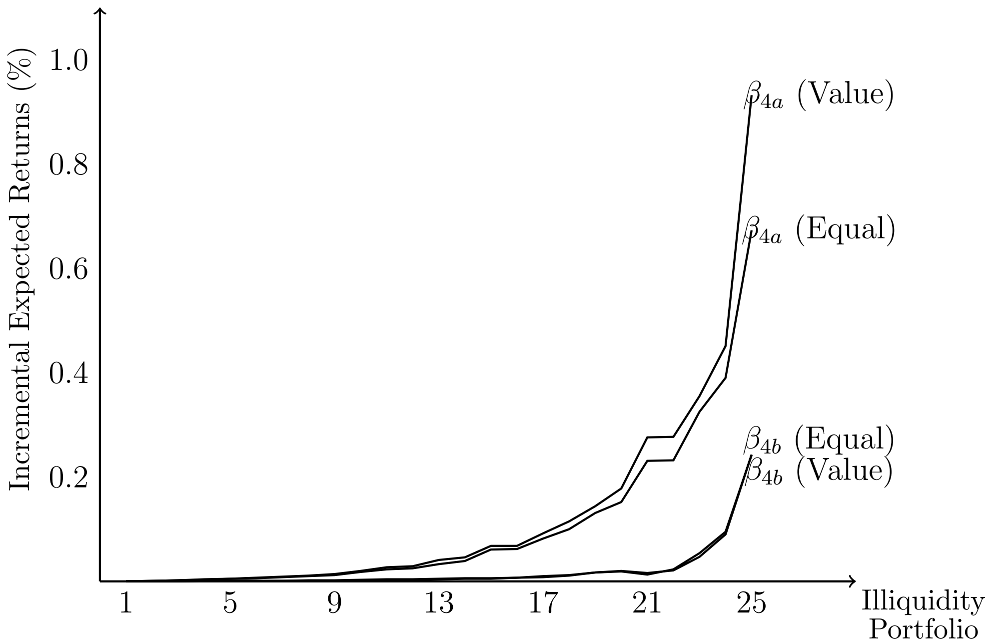
$$\beta_{4a} \text{ risk premium} = -12 \cdot 0.964 \cdot (-0.081 - 0) = 0.94\% \text{ (value-weighted)}$$

$$\beta_{4b} \text{ risk premium} = -12 \cdot 1.010 \cdot (-0.020 - 0) = 0.24\% \text{ (equal-weighted)}$$

$$\beta_{4b} \text{ risk premium} = -12 \cdot 0.964 \cdot (-0.021 - 0) = 0.24\% \text{ (value-weighted)}$$

Figure 1 depicts the above calculations for all illiquidity-sorted portfolios $\ell = 1, ..., 25$. Two points stand out. First, the premium resulting from covariation between illiquidity and business shocks always exceeds the premium resulting from covariation between illiquidity and financial shocks, although the absolute difference is small for stocks with above-median liquidity. Second, both types of premium exceed 0.2% only for portfolios in the lowest liquidity quintile; all other portfolios attract only economically insignificant premia.

Figure 1: β_{4a} and β_{4b} Risk Premia for Illiquidity-Sorted Portfolios



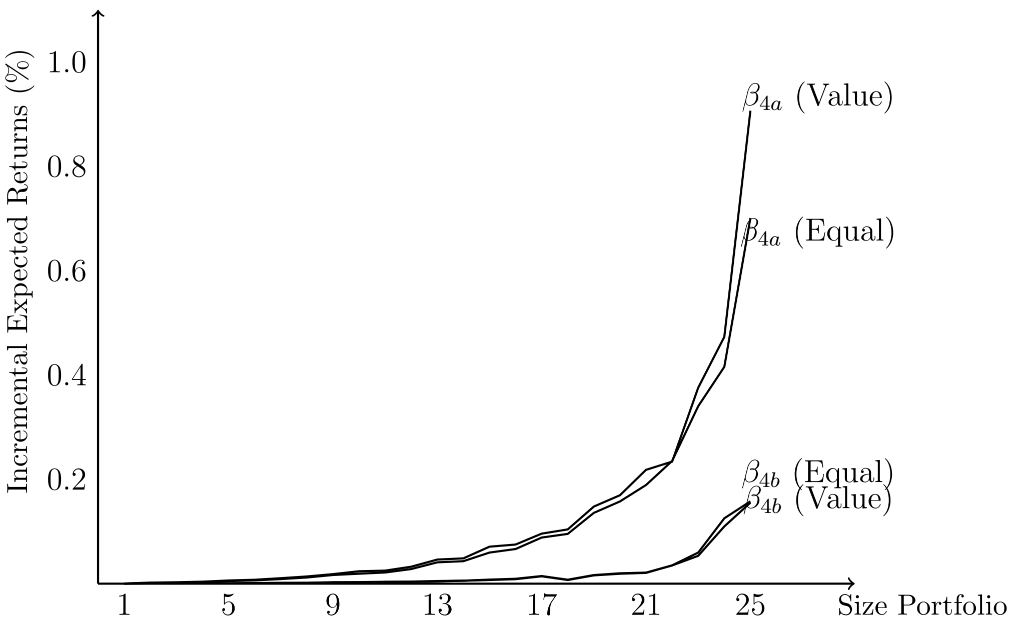
5 Additional Considerations

In the previous section, we found that the liquidity risk premium attributable to business shocks is substantially greater than the liquidity risk premium attributable to financial shocks, but that both premia are economically significant only for the most illiquid portfolios. In this section we consider some robustness and extension checks of these conclusions.

5.1 Alternative sorts

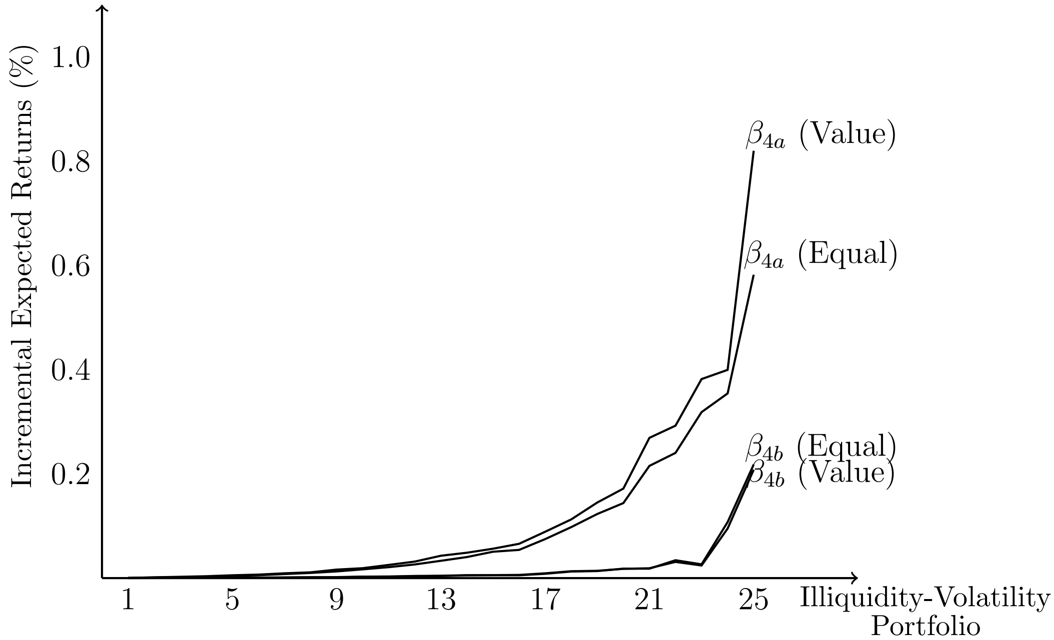
In section 4, we sorted portfolios according to a particular measure of liquidity C_p. Ex ante, other criteria are also defenisble and in this section, we consider two — size (market equity capitalization) and illiquidity volatility (standard deviation of portfolio illiquidity innovations). In these cases, the risk premia profiles appear in Figures 2 and 3 respectively.

Figure 2: β_{4a} and β_{4b} Risk Premia for Size-Sorted Portfolios



Unsurprisingly, given that illiquid portfolios are likely to contain small stocks that have high illiquidity volatility, sorting portfolios along the latter two dimensions has little effect on our original findings. The business shock premium is always greater than the financial shock premium, but neither is economically significant for most portfolios.

Figure 3: β_{4a} and β_{4b} Risk Premia for Portfolios Sorted on Illiquidity Volatility



5.2 Potential Overfitting Issues with $\eta_{d,t}$ and $\eta_{\pi t}$

In section 3.2, we estimate unexpected changes in market returns and market return variances using an ARMA model. The unexpected changes are defined as the difference between the fitted values of the ARMA model and actual value, and the model parameters are estimated from the entire sample. When estimated in this way, fitted values of expected market returns and variances are potentially subject to over-fitting and forward-looking bias.

To address this problem, we instead estimate expected market returns ($E_{t-1}[r_{Mt}]$) using a rolling-window moving average model, and expected market return variance ($E_{t-1}[\sigma^2_{t+1}]$) using an exponential weighted moving average (EWMA) model. The moving average model is widely used in the forecasting literature (Brock et al., 1992; Pesaran and Timmermann, 1995), while the EWMA estimator is a common approach to forecasting the conditional variance (Morgan, 1996), and is a special case of a generalized autoregressive conditional heteroskedascity (GARCH) model.

With these models, expected market returns and variance are estimated by:

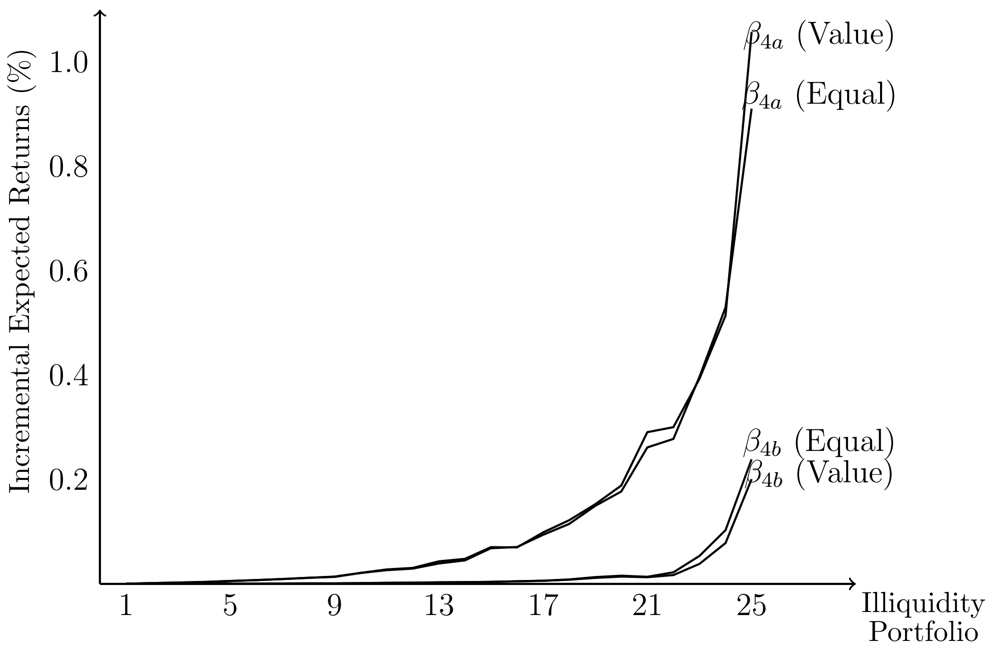
$$E_{t-1}\left[r_{Mt}\right] = \frac{1}{n}\sum_{i=1}^{n} r_{Mt-i} \tag{21}$$

$$E_{t-1}[\sigma^2_{t+1}] = (1-\delta)\sum_{i=1}^{m} \delta^i \left(\pi_{t-i} - \bar{\pi}\right)^2 \tag{22}$$

where n is the number of months in the moving average window, and $0 < \delta < 1$ is a constant decay (or smoothing) factor. We use $n = 60$ (5 years) for the rolling window moving average estimation and $\delta = 0.95$ for the exponential moving average estimation.

Equations (21) and (22) are used to estimate the market return and variance innovations, which in turn are used to estimate betas and risk premia for the illiquidity-sorted portfolios. The resulting risk premia profiles appear in Figure 4. The difference between the two premia is greater using this approach, and the premium associated with business shock liquidity risk is higher, but the overall picture is the same as in Figure 1. Any biases associated with our estimates of expected market returns and variances would seem to be relatively minor.

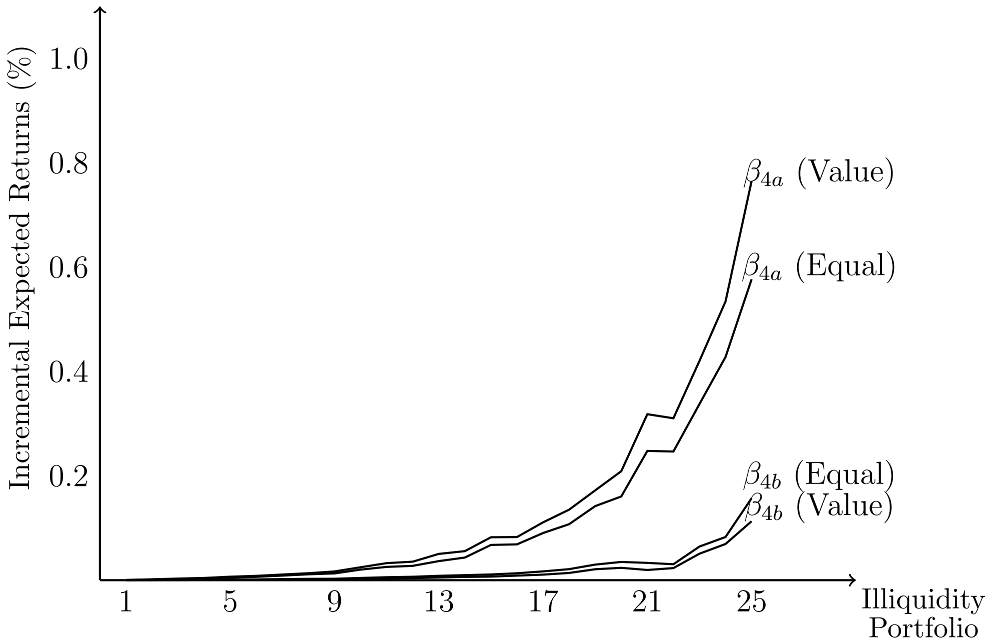
Figure 4: β_{4a} and β_{4b} Risk Premia for Portfolios Sorted on Illiquidity: MA and EWMA Innovations



5.3 1963-99 period

As noted in footnote 3, our estimates of liquidity costs C_p depend in part on parameters obtained by Acharya and Pedersen (2005) from 1963-99 data. If the values of these parameters have shifted over time, then our liquidity cost estimates, and hence our results, would be called into question. To address this issue, we therefore repeat our analysis using only 1963-99 data and summarize this in Figure 5. The outcome is almost identical to Figure 1, suggesting that the Acharya and Pedersen liquidity cost parameters have remained more or less stable in the post-1999 period, and therefore that their 1963-99 values are suitable for use in our extended period.

Figure 5: β_{4a} and β_{4b} Risk Premia for Portfolios Sorted on Illiquidity: 1963-99 Data



5.4 Three economic shocks

In section 2.2, we use the Campbell (1991) decomposition to split unexpected market returns into innovations resulting from (i) shocks to expectations about future discounted dividend growth and (ii) shocks to expectations about future market risk premia. We do this because it is most convenient (in the sense of requiring the weakest assumption about expectations) for the analysis in section 3.2. However, it is more usual to include the interest rate shock component either separately (Campbell and Mei, 1993) or together with the risk premium component (Campbell and Vuolteenaho, 2004). In our case, this is potentially important because we find that the largest component of systematic liquidity risk is attributable to what we call business shocks, rather than financial shocks, but as the former includes interest rates it also has a 'financial' dimension. This leaves open the possibility that the importance of business shocks is at least partly due to interest rate considerations.

To investigate this, first rewrite equation (4) as:

$$\begin{aligned} r_{Mt} - E_{t-1}\left[r_{Mt}\right] &\approx \Delta E_t\left[\sum_{j=0}^{\infty}\rho^j \Delta d_{t+j}\right] - \Delta E_t\left[\sum_{j=1}^{\infty}\rho^j \pi_{t+j}\right] - \Delta E_t\left[\sum_{j=1}^{\infty}\rho^j r_{ft+j}\right] \\ &= \eta_{gt} - \eta_{\pi t} - \eta_{r_{ft}} \end{aligned} \tag{23}$$

where $\eta_{gt} = \Delta E_t\left[\sum_{j=0}^{\infty}\rho^j \Delta d_{t+j}\right]$ is the weighted sum of shocks to expectations about future (undiscounted) dividend growth, and $\Delta E_t\left[\sum_{j=1}^{\infty}\rho^j r_{ft+j}\right]$ is the weighted sum of shocks to expectations about future short-term interest rates. Then making the additional assumption that the expected short-term interest rate is a constant, this becomes (remembering that r_{ft+1} is known at date t):

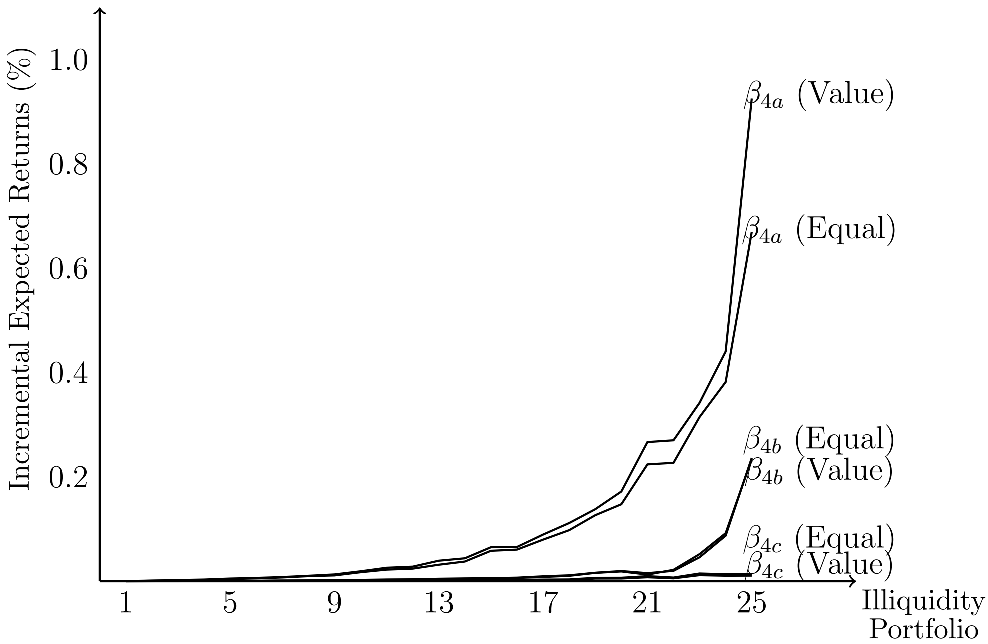
$$r_{Mt} - E_{t-1}\left[r_{Mt}\right] = \eta_{gt} - \frac{\rho}{1-\rho}\left[\,(E_t[\pi_{t+1}] - E_{t-1}[\pi_{t+1}]) + (r_{ft+1} - E_{t-1}[r_{ft+1}])\,\right]$$

Then, using (11):

$$r_{Mt} - E_{t-1}\left[r_{Mt}\right] = \eta_{dt} + \phi_1\left(\sigma^2_{t+1} - E_{t-1}[\sigma^2_{t+1}]\right) + \phi_2\left(r_{ft+1} - E_{t-1}[r_{ft+1}\right) \tag{24}$$

where $\phi_1 = \frac{-\rho\gamma}{1-\rho}$ and $\phi_2 = \frac{-\rho}{1-\rho}$. Estimating interest rate innovations with an ARMA model and otherwise proceeding as before yields Figure 6. The risk premia associated with dividend shocks are essentially the same as those previously observed for business shocks, while the risk premia associated with interest rate shocks are essentially zero, even for the most illiquid portfolios. Thus, liquidity risk appears to be primarily due to common variation in shocks to liquidity and aggregate dividends, partly due to common variation in shocks to liquidity and the market risk premium, and not at all due to common variation in shocks to liquidity and interest rates.

Figure 6: β_{4a} and β_{4b} Risk Premia for Portfolios Sorted on Illiquidity: 3-Way Decomposition



6 Conclusion

What aspects of liquidity risk matter for asset pricing? Acharya and Pedersen (2005) show that the primary contributor to the liquidity risk premium is common variation between shocks to liquidity and shocks to market portfolio returns. Extending this result, we find that this covariation is itself primarily due to common variation between shocks to liquidity and shocks to aggregate dividends. By contrast, common variation between shocks to liquidity and shocks to the market risk premium or interest rates attract only small premia. Overall, liquidity risk premia are generally small: even the premium attributable to dividend shocks is economically significant only for the top quintile of illiquid stocks.

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Appendix

1. Estimation of eqtns (15) and (16)

Table A1: ARMA(m,n) Estimation Results

Monthly ARMA models for logged market returns (r_{Mt}) and market variance (σ^2_{Mt}) using January 1964 – December 2017 data (648 months). The first four columns report results for logged market returns and the next four columns report results for logged market return variance:

$$r_{Mt} = \text{Constant} + \epsilon_{r_{Mt}} + \sum_{j=1}^{m} \psi_j r_{Mt-j} + \sum_{k=1}^{n} \theta_k \epsilon_{r_{Mt-k}}$$

$$\sigma^2_{Mt} = \text{Constant} + \epsilon_{\sigma^2_{Mt}} + \sum_{j=1}^{m} \psi_j \sigma^2_{Mt-1} + \sum_{k=1}^{n} \theta_k \epsilon_{\sigma^2_{Mt-k}}$$

Standard errors are in parentheses. AIC denotes the Akaike information criterion, BIC is the Bayesian information criterion, and ACF(1) is the first-order autocorrelation coef- ficient for the estimated residuals. LB(6) and LB(12) are Ljung-Box test statistics with 6 and 12 lags respectively. p-values for the Ljung-Box test are in square brackets.

	Logged Market Returns (r_{Mt})				Logged Market Return Variance (σ^2_{Mt})			
	ARMA (1,0)	ARMA (1,1)	ARMA (2,0)	ARMA (2,1)	ARMA (1,0)	ARMA (1,1)	ARMA (2,0)	ARMA (2,1)
Const.	0.014	0.014	0.014	0.014	0.002	0.002	0.002	0.002
	(0.003)	(0.003)	(0.003)	(0.003)	(0.000)	(0.000)	(0.000)	(0.000)
φ_1	0.159	-0.193	0.171	0.116	0.558	0.730	0.488	1.070
	(0.039)	(0.213)	(0.039)	(0.465)	(0.033)	(0.049)	(0.039)	(0.228)
φ_2			-0.070	-0.062			0.126	-0.205
			(0.039)	(0.085)			(0.039)	(0.146)
ϕ_1		0.365		0.055		-0.256		-0.589
		(0.202)		(0.466)		(0.071)		(0.216)
AIC	-1815.0	-1815.9	-1816.2	-1814.2	-5264.0	-5274.2	-5272.5	-5273.5
BIC	-1801.5	-1798.0	-1798.3	-1791.9	-5250.6	-5256.3	-5254.6	-5251.2
ACF(1)	0.011	-0.001	0.001	0.000	-0.070	0.007	-0.005	-0.003
LB(6)	6.168	3.305	3.003	2.997	16.177	4.419	5.806	3.125
	[0.405]	[0.770]	[0.809]	[0.809]	[0.013]	[0.620]	[0.445]	[0.793]
LB(12)	10.822	7.885	7.558	7.567	17.851	4.723	6.393	3.442
	[0.544]	[0.794]	[0.819]	[0.818]	[0.120]	[0.967]	[0.895]	[0.992]

2. Estimates of β_1, β_2, and β_3 for Illiquidity-Sorted Portfolios.

Table A2: Portfolio Betas and Characteristics for 25 Illiquidity Portfolios

Portfolio	β_1	β_2	β_3	β_1	β_2	β_3
Liquid	53.70	0.00	-0.37	86.02	0.00	-0.05
	(28.10)	(1.57)	(-5.09)	(53.17)	(2.08)	(-7.51)
3	65.29	0.00	-0.50	93.30	0.00	-0.06
	(39.96)	(6.75)	(-5.55)	(66.85)	(6.04)	(-8.30)
5	68.47	0.00	-0.53	94.29	0.00	-0.06
	(44.94)	(5.67)	(-6.12)	(53.88)	(5.52)	(-8.40)
7	75.53	0.00	-0.65	100.96	0.00	-0.07
	(40.32)	(6.59)	(-5.64)	(42.74)	(5.86)	(-8.48)
9	77.79	0.00	-0.67	101.65	0.00	-0.08
	(59.29)	(6.09)	(-6.46)	(39.67)	(6.08)	(-8.96)
11	76.21	0.00	-0.67	96.24	0.00	-0.07
	(52.77)	(6.13)	(-6.72)	(33.90)	(6.16)	(-8.76)
13	80.21	0.01	-0.69	101.60	0.00	-0.07
	(50.57)	(5.82)	(-6.15)	(33.72)	(5.24)	(-8.13)
15	81.88	0.01	-0.73	101.01	0.00	-0.08
	(56.70)	(6.02)	(-7.36)	(31.12)	(5.78)	(-8.06)
17	84.19	0.01	-0.80	101.83	0.00	-0.08
	(59.30)	(6.30)	(-7.50)	(29.76)	(5.75)	(-8.49)
19	86.44	0.03	-0.79	103.48	0.00	-0.08
	(46.58)	(6.46)	(-7.41)	(25.31)	(6.09)	(-8.31)
21	87.21	0.04	-0.84	102.65	0.01	-0.09
	(50.23)	(5.82)	(-6.99)	(23.93)	(5.65)	(-8.34)
23	81.10	0.08	-0.78	93.96	0.01	-0.08
	(35.33)	(6.79)	(-6.97)	(19.39)	(7.52)	(-7.35)
ILLIQ	74.38	0.21	-0.74	84.20	0.02	-0.07
	(27.58)	(6.05)	(-7.57)	(17.67)	(7.24)	(-6.86)
	Equal	Equal	Equal	Value	Value	Value

3. Estimation of λ

Table A3: Estimation of λ from 25 Illiquidity-Sorted Portfolios

This table reports cross-sectional liquidity-adjusted CAPM estimation results based on data for Jan 1964 - Dec 2017 (648 months). The regression model is either:
$r_{pt} - r_{ft} = \alpha + \mathcal{K}_1 E\left[C_{pt}\right] + \lambda\beta_{pt} + e_{pt}$, or:
$r_{pt} - r_{ft} = \alpha + \mathcal{K}_2 E\left[C_{pt} \cdot Turn_{pt}\right] + \lambda\beta_{pt} + e_{pt}$.
where $\beta_{pt} \equiv \beta_{1pt} + \beta_{2pt} - \beta_{3pt} - \beta_{4apt} - \beta_{4bpt}$ is the overall LCAPM beta of portfolio p in month t. The first specification allows $\mathcal{K}_1$ to differ from its theoretical value of 1 to allow for holding periods exceeding a month (portfolio average of 13.7 months in our sample). The second specification recognizes that holding periods may vary across portfolios of different liquidity, as suggested by Table 3. The estimated effects are the averages of estimates from 648 cross-sectional regressions using a GMM framework. Standard errors are computed using the Newey and West (1987) method with two lags. t-statistics are in parentheses. Adjusted R^2s are obtained from OLS regressions. Weight indicates whether the market portfolio used to estimate β_{pt} is equal or value-weighted. The weighted average estimates of λ are computed as $\sum_{i=1}^{4}\left(\lambda_i \times \frac{1/se(\lambda_i)}{\sum_{j=1}^{4} 1/se(\lambda_j)}\right)$.

	α	$\mathcal{K}_1$	$\mathcal{K}_2$	λ	Adj-R^2	Weight
1	-0.206	0.021		1.198	0.886	Equal
	(-0.707)	(1.159)		(2.623)		
2	-1.303	0.048		2.026	0.755	Value
	(-1.986)	(2.102)		(2.674)		
3	-0.173		0.773	1.133	0.886	Equal
	(-0.613)		(1.168)	(2.545)		
4	-1.143		1.691	1.828	0.755	Value
	(-1.871)		(2.126)	(2.591)		
5		0.025		0.936	0.996	Equal
		(1.196)		(3.673)		
6		0.043		0.726	0.987	Value
		(2.008)		(3.663)		
7			0.909	0.909	0.996	Equal
			(1.201)	(3.530)		
8			1.618	0.686	0.987	Value
			(2.091)	(3.515)		
Average (simple)				1.044		Equal
Average (simple)				1.347		Value
Average (Weighted)				1.010		Equal
Average (Weighted)				0.964		Value