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A Note on the Use of Partial Correlation Coefficients in Meta-Analyses

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Abstract: Meta-analyses in economics, business, and the social sciences commonly use partial correlation coefficients (PCCs) when the original estimated effects cannot be combined. This can occur, for example, when the primary studies use different measures for the dependent and independent variables, even though they are all concerned with estimating the same conceptual effect. This note demonstrates that analyses based on PCCs can produce different results than those based on the original, estimated effects. This can affect conclusions about the overall mean effect, the factors responsible for differences in estimated effects across studies, and the existence of publication selection bias. I first derive the theoretical relationship between Fixed Effects/Weighted Least Squares estimates of the overall mean effect when using the original estimated effects and their PCC transformations. I then provide two empirical examples from recently published studies. The first empirical analysis is an example where the use of PCCs does not change the main conclusions. The second analysis is an example where the conclusions are substantially impacted. I explain why the use of PCCs had different effects in the two examples.

Keywords: Meta-analysis, Publication bias, FAT-PET, Meta-regression analysis, Partial correlation coefficients

JEL Classifications: B41, C15, C18

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I. INTRODUCTION

Suppose a set of primary studies estimate how a variable *X* affects another variable *Y*. Let these estimated effects be represented by $\hat{\beta}_i$, and let μ be the mean population effect of *X* on *Y* across all studies. Meta-analyses commonly estimate a specification of the general form:

$$(1) \qquad \hat{\beta}_i = \mu + \varepsilon_i \; .$$

In the absence of publication bias, the estimate of μ provides an unbiased estimate of the mean population effect of *X* on *Y*.

However, in many research situations, it is not appropriate to aggregate the estimates, $\hat{\beta}_i$. This can arise when the primary studies use different measures for *X* and *Y*. For example, researchers may be interested in estimating how inward foreign direct investment (FDI) affects economic growth. Different studies could (and do) employ different measures of FDI (e.g., the ratio of FDI over GDP or the natural log of total FDI) and different measures of economic growth (e.g., growth in nominal GDP, growth in real GDP, growth in real GDP per capita).

When this happens, a common approach is to convert estimates into partial correlation coefficients, where

$$(2) \qquad PCC_i = \frac{t_i}{\sqrt{t_i^2 + df_i}} ,$$

and t_i and df_i are the *t*-statistic and degrees of freedom associated with the i^{th} estimated effect. Meta-analyses proceed by estimating specifications of the following general form:

$$(3) \qquad PCC_i = \theta + \eta_i \; .$$

$PCCs$ are widely used in economics, business and social science meta-analyses. For example, discussions of the use of $PCCs$ in meta-analyses can be found in Stanley & Doucouliagos (2012, pages 24-26), Ringquist (2013, pages 105-110), and Ellis (2010, pages 11f.). Recently published meta-analyses that use $PCCs$ include Bruno and Cipollina (2018); Iwasaki and Mizobata (2018); Cohen and Tubb (2018); Bijlsma, Kool, and Non (2018);

Churchill and Mishra (2018); Merkle and Phillips (2018); Churchill and Yew (2017); Valickova, Havranek, and Horvath (2015); Arestis, Chortareas, and Magkonis (2015); Wang and Shailer (2015); and Nataraj et al. (2014).

Despite the widespread use of $PCCs$, I am unaware of any study that discusses the relationship between the estimate of θ in Equation (3) and the estimate of μ in Equation (1). That is the purpose of this note. I derive the theoretical relationship between these two estimates and then provide two empirical examples from published studies where I show how they can differ.

The implied assumption in many meta-analyses that use $PCCs$ is that conclusions derived from estimating overall mean effects and meta-regressions, and from testing for publication bias, would also be valid for the untransformed effects $\hat{\beta}_i$ were it appropriate to use these. I show that this is not necessarily the case.

I. THE THEORETICAL RELATIONSHIP BETWEEN $\hat{\mu}$ AND $\hat{\theta}$

I illustrate the relationship between estimates of μ and θ using the Weighted Least Squares (WLS) estimator (Stanley & Doucouliagos, 2015). WLS produces identical coefficient estimates to Fixed Effects. I work with the former because it allows me to stay within the conventional Ordinary Least Squares (OLS) framework, which eases the exposition.

WLS estimation of μ in Equation (1) is identical to estimating Equation (4) below using OLS:

$$(4) \qquad \frac{\hat{\beta}_i}{SE_i} = t_i = \mu \cdot \left(\frac{1}{SE_i}\right) + \tilde{\varepsilon}_i,$$

where SE_i is the standard error of $\hat{\beta}_i$ from the primary study and $\tilde{\varepsilon}_i$ is the standardized transformation of ε_i. WLS estimation of θ in Equation (3) is given by OLS estimation of

$$(5) \qquad \frac{PCC_i}{SEPCC_i} = \theta \cdot \left(\frac{1}{SEPCC_i}\right) + \tilde{\eta}_i \ ,$$

where $SEPCC_i$ is the standard error of PCC_i, $\tilde{\eta}_i$ is the standardizedη_i, and

(6) $$SEPCC_i = \sqrt{\frac{1-PCC_i^2}{df_i}}\ .$$

The relationship between the WLS estimates of μ and θ becomes clear if we substitute Equation (6) into (5).

(7) $$\frac{PCC_i}{SEPCC_i} = \sqrt{\frac{PCC_i^2}{\left(\frac{1-PCC_i^2}{df_i}\right)}} = \sqrt{\frac{df_i \cdot PCC_i^2}{1-PCC_i^2}} = \sqrt{\frac{df_i \cdot PCC_i^2}{\left(1-\frac{t_i^2}{t_i^2+df_i}\right)}} = \sqrt{\frac{df_i \cdot PCC_i^2}{\left(\frac{t_i^2+df_i-t_i^2}{t_i^2+df_i}\right)}}$$

$$= \sqrt{PCC_i^2 \cdot (t_i^2 + df_i)} = \sqrt{\frac{t_i^2}{t_i^2+df_i} \cdot (t_i^2 + df_i)} = \sqrt{t_i^2} = t_i\ .$$

Note that t_i in Equation (7) is identical to t_i in Equation (4).

Substituting (7) into (4) yields

(8) $$t_i = \theta \cdot \left(\frac{1}{SEPCC_i}\right) + \tilde{\eta}_i\ ,$$

Equations (8) and (4) make clear that WLS estimation of μ consists of regressing t_i on $\left(\frac{1}{SE_i}\right)$ without a constant; while WLS estimation of θ consists of regressing t_i on $\left(\frac{1}{SEPCC_i}\right)$, also without a constant. Thus, meta-analyses using $PCCs$ will produce results similar to meta-analyses using the original estimates $\hat{\beta}s$ to the extent that $\left(\frac{1}{SEPCC_i}\right)$ is similar to $\left(\frac{1}{SE_i}\right)$.

II. TWO EMPIRICAL EXAMPLES

In this section I present examples from two meta-analyses, one where using PCC rather than $\hat{\beta}$ produces similar conclusions, and one where it does not.[1] Both meta-analyses estimate an "elasticity" of some outcome variable Y with respect to an explanatory variable X. In the case of de Linde Leonard, Stanley, & Doucouliagos (2014), the estimated elasticity measures the

[1] All the data and code to reproduce the results in this note are publicly available at *Harvard's Dataverse*: https://doi.org/10.7910/DVN/GXMOXS.

percent change in employment associated with a one-percent increase in the minimum wage. In the case of Havranek (2015), it measures the percent change in consumption growth associated with a one-percent increase in the net interest rate.

Both meta-analyses estimate variations of the following two regression equations:

(9.a) $\widehat{elasticity}_i = \alpha_0 + \alpha_1 SE_i + \epsilon_i$, and

(9.b) $\widehat{elasticity}_i = \beta_0 + \beta_1 SE_i + \sum_{k=2}^{K} \beta_k X_{ki} + \epsilon_i$,

where SE_i is the standard error corresponding to the i^{th} estimated elasticity, and the X_k are variables hypothesized to affect the estimated elasticity.

Equation (9.a) is the well-known FAT-PET specification (Funnel Asymmetry Test-Precision Effect Test). The FAT tests for publication selection bias, where rejection of $H_0: \alpha_1 = 0$ is evidence that publication selection biases the estimates in the meta-analyst's sample. The PET tests $H_0: \alpha_0 = 0$, where α_0 is the "publication selection bias-corrected" estimate of the overall mean elasticity. Meta-analysts interpret the estimates of Equation (9.a) to determine whether publication selection bias is present in their sample of estimates, and whether the overall mean effect is significantly different from zero.

Meta-analysts also frequently estimate specifications like Equation (9.b) to help explain different estimated effects. It is common to refer to the statistical significance of the respective coefficient estimates to identify key characteristics that contribute to different estimated effects across studies.[2]

In the examples below, I first reproduce the FAT-PET and meta-regression results reported in the respective studies. I then repeat the analysis, replacing $\widehat{elasticity}_i$ and SE_i in Equations (9.a) and (9.b) with their corresponding PCC_i and $SEPCC_i$ values. I compare the two sets of estimates and observe whether the use of PCCs affects the results.

[2] The pitfalls of using statistical significance to draw conclusions about practical importance are well-known. However, it is a common practice.

de Linde Leonard, Stanley, & Doucouliagos (2014). LS&D conduct a meta-analysis studying the effect of minimum wages on employment in the United Kingdom. They collect 236 minimum wage elasticities from twelve studies.[3] They use this sample to estimate Equation (9.a) using WLS.

I reproduce their results in the top panel of TABLE 1, Column (1).[4] The estimates of α_1 and α_0 are -0.42 and -0.008. Both estimates are statistically insignificant. This leads LS&D to conclude that there is "no statistical evidence of publication selection bias" and "no evidence of a genuine nonzero effect" (page 505). When I use the same sample, replace $\widehat{elasticity}_i$ and SE_i with PCC_i and $SEPCC_i$ (following Equations 2 and 6), and re-estimate Equation (9.a), I obtain estimates of α_1 and α_0 equal to -0.30 and -0.003. Both estimates remain statistically insignificant. Using PCC rather than $\widehat{elasticity}$ requires no change in LS&D's conclusions with respect to publication selection bias and the overall mean effect.

Panel B reports the results of estimating Equation (9.b). The individual variables (*Un, Toughness, Lag,* etc.) are described in LS&D. Column (1) reproduces LS&D's results.[5] They find that *Un, Toughness, AveYear,* and *SE* are all significant determinants of estimated elasticities. When $\widehat{elasticity}$ is replaced with PCC, *Un, Toughness,* and *SE* have the same signs as before and remain statistically significant. The only substantive differences are that *AveYear* is no longer significant, while *Region* is. In summary, using $PCCs$ rather than estimated elasticities in LS&D's sample produces very similar conclusions.

[3] LS&D also collect information on t-statistics and degrees of freedom that allow them to construct a parallel dataset of 710 *PCC* observations. They do not, however, directly compare elasticity and *PCC* results using the same observations, which is what I do in TABLE 1.
[4] I focus on LS&D's WLS estimates. They also report results using a random effects estimator.
[5] The results in Column (1) are intended to match LS&D's results in Column (1) of their Table 5. However, those results only use 231 observations, compared to 236 observations in their Table 1. The difference is due to the deletion of five outliers. Rather than try to reproduce their outlier strategy, I continued to use the sample of 236 observations. One consequence of this is that I was unable to include the variable *Adults*, which LS&D include in their meta-regression specification. Nevertheless, my Column (1) results reproduce their results very closely.

Havranek (2015). Havranek (2015) collects 2,735 estimates from 169 studies of the elasticity of intertemporal substitution in consumption. Like LS&D, he estimates FAT-PET and multiple meta-regression specifications.[6] These results are reported in TABLE 2, with the first column reproducing the results from Havranek. The second column reports the results of repeating the analysis, except that estimated elasticities and standard errors are replaced with their corresponding partial correlation equivalents.

Havranek's estimation of the FAT-PET specification produces $\hat{\alpha}_1 = 2.115$ and $\hat{\alpha}_0 = 0.0145$. $\hat{\alpha}_1$ is significant, while $\hat{\alpha}_2$ is not. Havranek concludes that these results suggest "strong selective reporting and zero underlying elasticity on average." When the FAT-PET specification is re-estimated using $PCCs$, the resulting estimates are $\hat{\alpha}_1 = -10.007$ and $\hat{\alpha}_0 = 0.390$. The latter *PET* estimate is statistically significant and indicates a large, mean intertemporal substitution effect. This contrasts with the very small, insignificant estimate of 0.0145 in the elasticity specification. Further, while both specifications produce significant *FAT* estimates, the elasticity results indicate positive publication selection – i.e., researchers and journals prefer to report larger elasticities – while the PCC results indicate the opposite.

The PCC specification also produces dramatically different multiple meta-regression results. Whereas the elasticity specification finds seven statistically significant determinants of estimated intertemporal substitution elasticities at the 5-percent level (*SE, Micro data, Asset holders, Epstein-Zin, Habit, Nonsep. durables, and Nonsep. tradables*), the PCC specification only finds two (*Nonsep. durables* and *Nonsep. tradables*). In summary, had Havranek used the PCC specification in his analysis, he would have come to very different conclusions with respect to the existence of publication selection bias, the economic significance of the elasticity

[6] Havranek uses a variety of estimation procedures. We reproduce his panel fixed effects results for the FAT-PET analysis. For the meta-regression analysis, we reproduce his WLS results that weight on the inverse of the number of estimates per study.

of intertemporal substitution in consumption, and the factors responsible for different estimates across studies.

Both LD&S and Havranek employ FAT-PET and multiple meta-regression specifications using elasticities to draw conclusions about their subjects. When elasticities are replaced with $PCCs$, LD&S' conclusions are largely unaffected, but Havranek's results change dramatically. The explanation lies with the relationship between $\left(\frac{1}{SEPCC}\right)$ and $\left(\frac{1}{SE}\right)$.

TABLE 3 reports Pearson correlations for $\left(\frac{1}{SEPCC}\right)$ and $\left(\frac{1}{SE}\right)$ for each of the studies. For LD&S, the two variables are highly correlated (correlation = 0.773). As a result, the results using elasticities and $PCCs$ are very similar. For the Havranek study, the corresponding correlation is -0.010 and statistically insignificant. It should therefore not be surprising that the corresponding analyses using $PCCs$ produce different results than those based on elasticities.

III. CONCLUSION

It is common in economics, business and social science meta-analyses to conduct analyses using partial correlation coefficients ($PCCs$) when it is inappropriate to directly combine estimated effects from different studies. The unstated assumption is that the conclusions from these analyses are applicable to the estimated effects. For example, if the subject of interest is the elasticity of economic growth with respect to FDI, and the meta-analyst draws conclusions based on analyses using $PCCs$, it is generally assumed that these conclusions would also be valid if one were able to directly use the respective elasticity estimates.

This note shows that this need not be the case. Analyses using $PCCs$ will produce similar results only when the inverses of the standard errors of the $PCCs$ are closely related to those of the estimated effects. As a result, meta-analysts should be cautious in extrapolating results based on PCC analyses. Further, if the estimated effects, rather than the $PCCs$, are directly of interest, meta-analysts may decide to be more tolerant of working directly with estimated effects even though the respective estimates are not perfectly compatible.

HIGHLIGHTS

- Meta-analyses in economics, business, and the social sciences commonly use partial correlation coefficients (*PCCs*)
- I demonstrate that analyses based on *PCCs* can produce different results than those based on the original, estimated effects
- This can affect conclusions about the overall mean effect, the factors responsible for differences in estimated effects across studies, and the existence of publication selection bias
- I provide two examples from recently published studies to illustrate when *PCCs* are likely to produce different results
- Meta-analysts should be cautious in using results based on $PCCs$ to make conclusions about the original, estimated effects

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TABLE 1
Comparing Meta-Analysis Results Using Elasticities with *PCCs:*
de Linde Leonard, Stanley, & Doucouliagos (2014)

A. FAT/PET Results

	Elasticity (1)	*PCC* (2)
FAT (α_1)	-0.42 (-0.70)	-0.30 (-0.46)
PET (α_0)	-0.008 (-0.73)	-0.003 (-1.02)
Obs	236	236

NOTE: Compare Column (1) with Column (1) in TABLE 1 of de Linde Leonard, Stanley, & Doucouliagos, 2014 page 505). Values in parentheses are t-statistics using cluster robust standard errors (by study). *, **, and *** indicate statistical significance at the 10-, 5-, and 1-percent levels, respectively.

B. Multiple Meta-Regression Results

	Elasticity (1)	*PCC* (2)
Un	-0.437*** (-7.44)	-0.293*** (-7.01)
Toughness	0.567*** (3.54)	0.373*** (4.87)
Lag	0.034 (0.37)	-0.014 (-0.85)
Published	-0.095 (-0.90)	-0.032 (-0.83)
Time	-0.082 (-0.69)	-0.046 (-1.24)
AveYear	0.011** (2.99)	0.003 (0.66)
Panel	0.017 (0.10)	0.114 (0.57)
Double	-0.046 (-0.49)	0.001 (0.03)
Region	0.003 (0.61)	-0.002*** (-3.32)
SE	-0.964*** (-3.16)	-1.765** (-2.96)
Obs	236	236

NOTE: Compare Column (1) with Column (1) in TABLE 5 of de Linde Leonard, Stanley, & Doucouliagos, 2014 page 510). See LS&D's Table 3 (page 508) for definitions of the variables. Values in parentheses are t-statistics using cluster robust standard errors (by study). *, **, and *** indicate statistical significance at the 10-, 5-, and 1-percent levels, respectively.

TABLE 2
Comparing Meta-Analysis Results Using Elasticities with *PCCs:* Havranek (2015)

A. FAT/PET Results

	Elasticity (1)	*PCC* (2)
FAT **(α_1)**	2.115*** (0.205)	-10.007** (5.047)
PET **(α_0)**	0.0145 (0.00881)	0.390** (0.158)
Obs	2,735	2,735

NOTE: Compare Column (1) with Column (1) in Table 1 of Havranek, 2015, page 1185. Values in parentheses are cluster robust standard errors (by study). *, **, and *** indicate statistical significance at the 10-, 5-, and 1-percent levels, respectively.

B. Multiple Meta-Regression Results

	Elasticity (1)	*PCC* (2)
SE	1.926*** (0.251)	-0.827 (1.712)
Micro data	0.209*** (0.0308)	-0.295 (0.191)
Asset holders	0.174*** (0.0365)	-0.020 (0.014)
Epstein-Zin	-0.0200*** (0.00655)	0.171* (0.099)
Habits	0.425*** (0.0671)	0.036 (0.054)
Nonsep. durables	0.0320*** (0.00324)	0.347** (0.167)
Nonsep. public	0.0709 (0.0871)	-0.090 (0.111)
Nonsep. tradables	0.358*** (0.0456)	0.491*** (0.158)
Constant	0.00512 (0.00322)	0.329* (0.198)
Obs	2,735	2,735

NOTE: Compare Column (1) with Column (2) in Table 2 of Havranek, 2015, page 1193. Variables are defined in Table A.2 in Havranek, page 1200. Values in parentheses are cluster robust standard errors (by study). *, **, and *** indicate statistical significance at the 10-, 5-, and 1-percent levels, respectively.

TABLE 3
Correlation between $\left(\frac{1}{SEPCC}\right)$ and $\left(\frac{1}{SE}\right)$

	Study	
	LD&S (2014)	**Havranek (2015)**
Correlation between $\left(\frac{1}{SEPCC}\right)$ and $\left(\frac{1}{SE}\right)$	0.773 (*p-value* = 0.000)	-0.010 ((*p-value* = 0.606)
Obs	236	2,735