DEPARTMENT OF ECONOMICS AND FINANCE

SCHOOL OF BUSINESS AND ECONOMICS

UNIVERSITY OF CANTERBURY

CHRISTCHURCH, NEW ZEALAND

New Results on Entrepreneurship and Risk

Claudio A. Bonilla
Marcos Vergara
Richard Watt

WORKING PAPER

No. 20/2019

Department of Economics and Finance
School of Business
University of Canterbury
Private Bag 4800, Christchurch
New Zealand

WORKING PAPER No. 20/2019

New Results on Entrepreneurship and Risk

Claudio A. Bonilla[1†]
Marcos Vergara[2]
Richard Watt[3]

December 2019

Abstract: In this paper we study two decisions made by an individual. First, the decision to transit from paid and secure employment into risky entrepreneurship and second, the decision about the size or scale of the venture for transitioned entrepreneurs. In doing so, we focus on the risk attitudes and characteristics of the decision makers to analyze the effects of greater risk aversion, wealth increases and stochastic dominant shifts in the distribution of results. Interesting results arise for our comparative static results, where risk aversion, the DARA property and an upper bound for absolute risk aversion coefficient play the key roles in our results.

Keywords: Entrepreneurship, Risk Aversion, DARA.

JEL Classifications: D81; L26.

[1] School of Economics and Business, University of Chile, CHILE

[2] Faculty of Economics and Business, Universidad del Desarrollo, CHILE

[3] Department of Economics and Finance, University of Canterbury, NEW ZEALAND

†Corresponding author: Claudio Bonilla, email: cbonilla@fen.uchile.cl

New Results on Entrepreneurship and Risk

1 Introduction

The theory of entrepreneurship contains an ample spectrum of possible research questions to be tackled from the economic perspective, such as, for example: the economic reasons why a worker may decide to become an entrepreneur; the way an entrepreneur may finance a new venture; the effects of entrepreneurship on economic development; or the importance of legal and economic institutions in the entrepreneurial process. For the most part, these economic analyses are based on the concepts of uncertainty, financial constraints and potential bankruptcy, and the risk characteristics associated with the preferences of entrepreneurs and workers. The reason for this is very simple: uncertainty and risk are key elements in the development of new ventures.

Knight (1921) was the first to develop the idea of the connection between entrepreneurship and risk attitudes. Knight's idea was subsequently formalized and translated into economics models by Kanbur (1979) and Kihlstrom and Laffont (1979), and since then, it has been a core component of the economic theory of entrepreneurship (Vereshchagina and Hopenhayn, 2009; Koudstaal et al., 2015). The main idea behind this theory is that the wealthy are, on average, **less risk averse** than the poor because well-behaved utility functions present the property of decreasing absolute risk aversion (DARA) and therefore, the wealthy are more prone to starting risky ventures. Furthermore, there is a second reason why (but this line of research is out of the scope of this paper) the wealthy are more likely to become entrepreneurs, and that is because they face fewer financial constraints in securing capital (Evan and Jovanovic, 1989). Consequently, we should expect to observe mostly wealthy people choosing the occupation of entrepreneurship. The poor, in contrast, are more likely to become employees for a certain, fixed wage for both of the reasons mentioned above.

Most of the recent literature that uses microeconomic models of entrepreneurship based on risk attitudes builds on the DARA assumption. Examples of the importance of this assumption are the works of Cressy (2000), van Praag and Cramer (2001), Cramer et al. (2002), Hartog et al. (2002), Kan and Tsai (2006), Ahn (2009), Caliendo et al. (2010), and Hvide and Panos (2014) among others. Our paper stands on this literature to go deeper into the problem of self-selection of occupations and entrepreneurship under risk.

In this paper we focus on two specific aspects of the relation between entrepreneurship and risk. We first study the effects of risk on the scale (the size) of a new venture when an individual has already decided to become an entrepreneur. We then analyze the decision to transit from secure employment into entrepreneurship. We will study the effect of different levels of risk aversion on the size of the venture, the effect of initial wealth on the occupational self-selection decision and also, the effect of changes on the stochastic distribution of results on entrepreneurship.

Interesting results arise from our analysis. First, the commonly used assumptions of risk aversion and DARA, so frequently found in the literature of entrepreneurship, are not enough to secure the transition from secure employment into entrepreneurship as wealth increases. We will show that the sufficient condition also requires that the Arrow-Pratt absolute risk aversion coefficient to be smaller than an upper bound. Likewise, our results also show that the same condition on risk aversion is required to guarantee that an increase in risk free wealth will generate a larger scale of business for any agent currently acting as an entrepreneur, and for an increase in risk aversion (with constant wealth) to decrease the optimal scale of the business. The effects generated by stochastic dominance for the optimal scale of the entrepreneur's business are more complex, and (for second-order stochastic dominance) also require an upper bound on prudence.

In the next section we present the basic model of self-selection of occupation. Section 3 studies the scale effect and provides our first results. Section 4 studies the transition effect and presents our second group of results. Section 5 present some results based on stochastic dominant shifts os the distributions. Finally section 6 concludes.

2 The Basic Model of Self-Selection of Occupations

This economy is characterized by a single-good stochastic production function $f(L, \theta)$, where L is the labor hired and θ is a random variable indexing the state of the world and representing uncertainty in the model. We can think of θ as a random productivity shock that affects positively the production function and its marginal production, and therefore $f(L, \theta)$ and its first derivative $f_L(L, \theta)$ are increasing in θ, and the production function satisfies $f_{LL} < 0 < f_L$. Inada conditions are assumed to hold and therefore, an interior solution to the problem is expected. We assume that the price at which the good in question is sold is equal to 1, so that $f(L, \theta)$ also doubles as the revenue function.[1]

Assume (for the time being) that agents have identical preferences but differ in the initial level of wealth. They have the utility function $u(y)$, where y is the realized income. The utility function satisfies $u'' < 0 < u'$ and prudence ($u''' > 0$) coined by Kimball (1990) and widely used in models of precautionary savings and precautionary effort like Eeckhoudt et al. (2012) or Wang and Li (2014), and in models showing higher-order risk attitudes like Menegatti (2014) or Eeckhoudt et al. (2016).

Agents vary in the amount of initial wealth a, the distribution of which is exogenous. The agents have to choose between two occupations. They can become workers and earn a certain wage w (their total final wealth in this case would be $a + w$), or

[1] An alternative assumption is that θ measures the price of the good.

they can become entrepreneurs, hiring L units of labor and earning the residual profit from a stochastic production function, which is denoted as $y(\theta) = f(L, \theta) - wL + a$, where the stochastic component is our random variable θ.

Each agent takes w as given and chooses the occupation that offers the highest utility. This result is a competitive equilibrium that translates into a partition of the set of agents into a set of workers and a set of entrepreneurs.

Let $V_E(a) = E_\theta u(f(L, \theta) - wL + a))$ be the expected utility function of the entrepreneur for a given wealth level a. And let $V_W(a) = u(w + a)$ be the utility function of the worker for a given wealth level a. In equilibrium, there is a wealth level $\bar{a}$ at which an individual is indifferent to any of the two occupations, i.e., the level of utility is the same whether the individual is a worker or an entrepreneur. We will call this decision maker the marginal or the indifferent entrepreneur. Therefore, at $\bar{a}$ we have:

$$E_\theta u(f(L^*(w, \bar{a}), \theta) - wL^*(w, \bar{a}) + \bar{a})) = u(w + \bar{a}) \tag{1}$$

where $L^*(w, \bar{a})$ comes from the expected utility maximization of the entrepreneur, i.e., $L^*(w, \bar{a}) = \underset{\{L\}}{Argmax} \{E_\theta u(f(L, \theta) - wL + \bar{a}))\}$, which is obtained by the following first-order condition:

$$E_\theta[u'(f(L^*(w, a), \theta) - wL^*(w, a) + \bar{a})(f_L(L^*(w, a), \theta) - w)] = 0 \tag{2}$$

Note that w in (1) corresponds to the certainty equivalent of the entrepreneur's optimal random income for the marginal agent $\bar{a}$. Then, by Jensen inequality we know that for any risk-averse agent ($u'' < 0$), the certainty equivalent (w) is smaller than the expected value of the random variable, i.e., $w < E_\theta f(L^*(w,\bar{a}),\theta) - wL^*(w,\bar{a})$ and therefore, in our entrepreneurial context, we can say that for the marginal or indifferent entrepreneur, the expected value of his residual profits is greater than the wage received by being an employee.

With this general set up we will study the conditions under which the indifferent decision maker transit from secure employment into entrepreneurship as wealth increases. i.e., the conditions under which the slope (under a) of equation (1) at $\bar{a}$ satisfy:

$$V'_E(\bar{a}) > V'_W(\bar{a}) \tag{3}$$

The graph below represents equation (3)

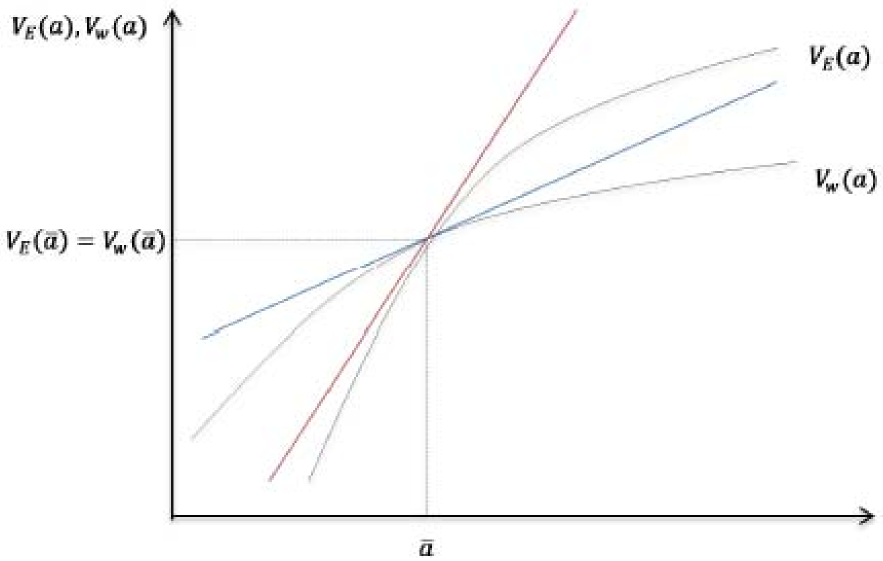

Figure 1: Marginal utilities of employee and entrepreneur at $\overline{a}$

But before we do that, let us focus now in a related but somewhat different problem, which is the problem of the scale of the entrepreneurial venture, i.e. we first study the choice of the size of the projects, as measured by $L^*(w, a)$, for decision makers that have already transitioned into entrepreneurship.

3 The Scale Effect

Assume that the decision maker has committed to being an entrepreneur (i.e. he has already transitioned), and so must resolve the following entrepreneurial problem:

$$\underset{\{L\}}{Max}\ E_\theta u(f(L,\theta) - wL + a) \tag{4}$$

The first-order condition for the problem is given by equation (2) evaluated at any wealth level a. The second-order condition for this maximum is satisfied by the assumed concavity of both u and f in L. The resulting optimal demand for labor is a function of a, written as $L^*(w, a)$. The scale effect is to show that $L^*_a(w, a) > 0$, so that wealthier decision makers embark upon a larger scale project (as measured by employment hired in the project).

To begin with, assume that the decision maker is risk-neutral. In that case, $u'(f(L, \theta) - wL + a)$ is a constant, and the first-order condition would read as

$$E_\theta f_L(L,\theta) - w = 0 \tag{5}$$

Notice that in this case, the optimal scale of the enterprise is independent of a. let us denote the optimal employment under risk-neutrality as $L_0^*(w)$. Clearly, this optimum simply maximizes the expected profit of the project for the risk neutral decision maker.

Now assume risk aversion, so the original first-order condition (equation (2)) applies. Since, for any two random variables $\widetilde{x}$ and $\widetilde{y}$ it holds that $E\widetilde{x}\widetilde{y} = E\widetilde{x}E\widetilde{y} + cov(\widetilde{x},\widetilde{y})$, the first-order condition can be expressed as

$$E_\theta u'(f(L,\theta)-wL+a)E_\theta\left(f_L(L,\theta)-w\right)+Cov\left(u'(f(L,\theta)-wL+a),(f_L(L,\theta)-w)\right)=0 \tag{6}$$

Consider first the term $Cov\left(u'(f(L,\theta)-wL+a),(f_L(L,\theta)-w)\right)$. By the initial assumptions of our model, an increase in θ will increase $f_L(L,\theta)$, so it will increase $f_L(L,\theta)-w$. But an increase in θ will also increase $f(L,\theta)$, and so will increase $f(L,\theta)-wL+a$, and thus it will decrease $u'(f(L,\theta)-wL+a)$. This implies that $Cov\left(u'(f(L,\theta)-wL+a),(f_L(L,\theta)-w)\right)<0$, and consequently $E_\theta u'(f(L,\theta)-wL+a)E_\theta\left(f_L(L,\theta)-w\right)>0$. Finally, since marginal utility is everywhere positive (i.e. $E_\theta u'(f(L,\theta)-wL+a)>0$), this implies that at $L^*(w,a)$ for a risk-averse decision maker, we must have

$$E_\theta f_L(L^*(w,a),\theta) > w = E_\theta f_L(L_0^*(w),\theta) \tag{7}$$

Now, since $f_{LL}<0$, the only way that we can get inequality (7) is that $L^*(w,a) < L_0^*(w)$. So, any risk-averse decision maker will invest in a smaller scale project than a risk-neutral decision maker will (so long as both decide to become entrepreneurs rather than work as employees). This previous result will be presented as a lemma.

Lemma 1. *A risk averse entrepreneur will invest in smaller projects than a risk neutral entrepreneur, i.e.* $L^*(w, a) < L_0^*(w)$.

3.1 Scale and greater risk aversion

The above result that risk averse investors will develop a smaller scale project than risk neutral investors is suggestive that as risk aversion increases, the scale of the investment will decrease. We can study the effect of greater risk aversion by comparing the optimal scale under a given utility function with that corresponding to a more risk-averse utility function and that is what we present in our next proposition.

Proposition 1. *When the absolute risk aversion coefficient is small enough, increases in risk aversion induce smaller ventures for the transitioned entrepreneur.*

Proof. Let us define the utility function as $u(y)$, and denote the optimal scale by $L_u^*(w, a)$, where that optimal scale is the solution to the corresponding first-order condition;

$$E_\theta \left[u'(f(L_u^*(w, a), \theta) - wL_u^*(w, a) + a) \left(f_L(L_u^*(w, a), \theta) - w \right) \right] = 0 \qquad (8)$$

Consider a second utility function, $v(y) \equiv s(u(y))$, where $s''(u(y)) < 0 < s'(u(y))$, so that $v(y)$ is a concave transformation and therefore, more risk-averse than $u(y)$. In this case, the optimal scale $L_v^*(w,a)$ is defined by the following first-order condition;

$$E_\theta\left[v'(f(L_v^*(w,a),\theta) - wL_v^*(w,a) + a)\left(f_L(L_v^*(w,a),\theta) - w\right)\right] = 0 \tag{9}$$

and given that $v(y) = s(u(y))$, first-order condition (9) translates into

$$E_\theta\left[s'\left(u(f(L_v^*(w,a),\theta) - wL_v^*(w,a) + a)\right)u'(f(L_v^*(w,a),\theta) - wL_v^*(w,a) + a)\left(f_L(L_v^*(w,a),\theta) - w\right)\right] = \tag{10}$$

To see how $L_v^*(w,a)$ compares with $L_u^*(w,a)$, substitute $L_u^*(w,a)$ into the first-order condition (10) to obtain

$$E_\theta\left[s'\left(u(f(L_u^*(w,a),\theta) - wL_u^*(w,a) + a)\right)u'(f(L_u^*(w,a),\theta) - wL_u^*(w,a) + a)\left(f_L(L_u^*(w,a),\theta) - w\right)\right] \tag{11}$$

If this expression (11) is negative, it meas that $L_v^*(w,a) < L_u^*(w,a)$ and therefore, as risk aversion increases, the scale of the project will decrease. This is simply because expected utility with $v(y)$ is strictly concave and has a maximum at $L_v^*(w,a)$ - where condition (11) equals zero -, and at any point to the right of the maximum ($L_v^*(w,a) < L_u^*(w,a)$) condition (11) must be negative. To see if this is the case define:

$$k(L,\theta) \equiv u'(f(L,\theta) - wL + a)\left(f_L(L,\theta) - w\right)$$

and calculate

$$\frac{\partial k(L,\theta)}{\partial\theta} = u''(f(L,\theta) - wL + a)\left(f_L(L,\theta) - w\right)f_\theta(L,\theta) + u'(f(L,\theta) - wL + a)f_{L\theta}(L,\theta) \tag{12}$$

Under the initial assumptions of increasing concave utility $u(y)$, $f_\theta(L,\theta) > 0$, and $f_{L\theta}(L,\theta) > 0$, along with the result shown above that $f_L(L,\theta) - w > 0$ at any risk-averse optimum, the first-term of (12) is negative and the second term is positive. Let's assume that overall, this derivative is positive, so that $k(L,\theta)$ increases with θ. We will revisit this assumption below.

The expression that we need to sign is now more simply written as

$$E_\theta \left[s'\left(u(f(L_u^*(w,a),\theta) - wL_u^*(w,a) + a)\right) k(L_u^*(w,a),\theta)\right] \tag{13}$$

Consider first the slope of $s'\left(u(f(L_u^*(w,a),\theta) - wL_u^*(w,a) + a)\right)$ in θ. This derivative is

$$s''\left(u(f(L_u^*(w,a),\theta) - wL_u^*(w,a) + a)\right) u'(f(L_u^*(w,a),\theta) - wL_u^*(w,a) + a) f_\theta(L_u^*(w,a),\theta) < 0 \tag{14}$$

Second, we know from the first order condition (8) that $E_\theta k(L_u^*(w,a),\theta) = 0$, so there must exist an $\overline{\theta}$ such that $k(L_u^*(w,a),\overline{\theta}) = 0$. Our assumption that the slope of k is positive implies that this $\overline{\theta}$ is unique, and that

$$\begin{aligned} k(L_u^*(w,a),\theta) &< 0 \text{ for all } \theta < \overline{\theta} \\ k(L_u^*(w,a),\theta) &> 0 \text{ for all } \theta > \overline{\theta} \end{aligned} \tag{15}$$

Without loss of generality, assume further that θ takes values between $-\infty$ and ∞ and that the probability density distribution of θ is given by $g(\theta)$. Given that, our problem is reduced to putting a sign to the following expression

$$\int_{-\infty}^{\infty} s'\left(u(f(L_u^*(w,a),\theta) - wL_u^*(w,a) + a)\right) k(L_u^*(w,a),\theta) g(\theta) d\theta \tag{16}$$

Since $s'(u(y))$ decreases with θ we have

$$\int_{-\infty}^{\overline{\theta}} s'\left(u(f(L_u^*(w,a),\theta) - wL_u^*(w,a) + a)\right) g(\theta)d\theta \quad > \quad \int_{-\infty}^{\overline{\theta}} s'\left(u(f(L_u^*(w,a),\overline{\theta}) - wL_u^*(w,a) + a)\right) g(\theta)d\theta \tag{17}$$

and

$$\int_{\overline{\theta}}^{\infty} s'\left(u(f(L_u^*(w,a),\theta) - wL_u^*(w,a) + a)\right) g(\theta)d\theta \quad < \quad \int_{\overline{\theta}}^{\infty} s'\left(u(f(L_u^*(w,a),\overline{\theta}) - wL_u^*(w,a) + a)\right) g(\theta)d\theta \tag{18}$$

Multiply through by $k(L_u^*(w,a),\theta)$ which, given (15), we know is negative in the top inequality and positive in the bottom one:

$$\int_{-\infty}^{\overline{\theta}} s'\left(u(f(L_u^*(w,a),\theta) - wL_u^*(w,a) + a)\right) k(L_u^*(w,a),\theta)g(\theta)d\theta \quad < \quad \int_{-\infty}^{\overline{\theta}} s'\left(u(f(L_u^*(w,a),\overline{\theta}) - wL_u^*(w,a) + a)\right) k(L_u^*(w,a),\theta)g(\theta)d\theta \tag{19}$$

and

$$\int_{\overline{\theta}}^{\infty} s'\left(u(f(L_u^*(w,a),\theta) - wL_u^*(w,a) + a)\right) k(L_u^*(w,a),\theta)g(\theta)d\theta \quad < \quad \int_{\overline{\theta}}^{\infty} s'\left(u(f(L_u^*(w,a),\overline{\theta}) - wL_u^*(w,a) + a)\right) k(L_u^*(w,a),\theta)g(\theta)d\theta \tag{20}$$

Finally, summing up the two left-hand sides together and the two right-hand sides together of (19) and (20) we get:

$$\int_{-\infty}^{\infty} s'\left(u(f(L_u^*(w,a),\theta) - wL_u^*(w,a) + a)\right) k(L_u^*(w,a),\theta)g(\theta)d\theta \quad < \quad \int_{-\infty}^{\infty} s'\left(u(f(L_u^*(w,a),\overline{\theta}) - wL_u^*(w,a) + a)\right) k(L_u^*(w,a),\theta)g(\theta)d\theta \tag{21}$$

And operating on the right-hand side of (21) we get

$$\begin{aligned}
&\int_{-\infty}^{\infty} s'\left(u(f(L_u^*(w,a),\overline{\theta}) - wL_u^*(w,a)+a)\right) k(L_u^*(w,a),\theta)g(\theta)d\theta \\
&= s'\left(u(f(L_u^*(w,a),\overline{\theta}) - wL_u^*(w,a)+a)\right) \int_{-\infty}^{\infty} k(L_u^*(a,w),\theta)g(\theta)d\theta \\
&= s'\left(u(f(L_u^*(w,a),\overline{\theta}) - wL_u^*(w,a)+a)\right) E_\theta k(L_u^*(a,w),\theta) = 0 \qquad (22)
\end{aligned}$$

The final equality comes from the fact that $E_\theta k(L_u^*(w,a),\theta)$ is just the first-order condition for the problem with utility function $u(y)$. This means that

$$\int_{-\infty}^{\infty} s'\left(u(f(L_u^*(w,a),\theta) - wL_u^*(w,a)+a)\right) k(L_u^*(w,a),\theta)g(\theta)d\theta < 0$$

And that is exactly the required result we are looking for. In short, the assumption that $k(L,\theta)$ increases with θ is a sufficient condition for the conclusion that $L_v^*(w,a) < L_u^*(w,a)$, that is, an increase in risk aversion decreases the optimal scale of the entrepreneurial project.

But now we need to consider the implication behind the assumption that $k(L,\theta)$ increases with θ? Going back to the expression above for $\frac{\partial k(L,\theta)}{\partial \theta}$, we observe that assuming it is positive implies that

$$u''(f(L,\theta)-wL+a)\left(f_L(L,\theta)-w\right)f_\theta(L,\theta)+u'(f(L,\theta)-wL+a)f_{L\theta}(L,\theta) \;>\; 0 \quad (23)$$

That is

$$u'(f(L,\theta)-wL+a)f_{L\theta}(L,\theta) \;>\; -u''(f(L,\theta)-wL+a)\left(f_L(L,\theta)-w\right)f_\theta(L,\theta) \quad (24)$$

which in turns implies that

$$\frac{f_{L\theta}(L,\theta)}{\left(f_L(L,\theta)-w\right)f_\theta(L,\theta)} \;>\; -\frac{u''(f(L,\theta)-wL+a)}{u'(f(L,\theta)-wL+a)} \equiv A(y(w,a),\theta) \qquad (25)$$

Where $A(y(w,a),\theta)$ is the Arrow-Pratt absolute risk aversion coefficient and in consequence, for the scale of the venture to be decreasing under higher levels of risk aversion, we need that the absolute risk aversion coefficient to be small enough. $\square$

3.2 Scale and Effect of an increase in wealth a

Now, we are in a good position to study the effect of wealth increases into the scale of the ventures for transitioned entrepreneurs. Let us define

$$h(L^*,w,a) \equiv E_\theta k(L^*,\theta) = E_\theta u'(f(L^*,\theta) - wL^* + a)\,(f_L(L^*,\theta) - w)$$

From the first-order condition (8) we know that $h(L^*,w,a) = 0$, then applying the implicit function theorem, we know that

$$L_a^*(w,a) = -\frac{h_a(L^*,w,a)}{h_L(L^*,w,a)} \tag{26}$$

But since $h_L(L^*,w,a) < 0$ from the second order condition of the original problem, $L_a^*(w,a)$ has the same sign as $h_a(L^*,w,a)$, where

$$h_a(L^*,w,a) = E_\theta u''\,(f(L^*,\theta) - wL^* + a)\,(f_L(L^*,\theta) - w) \tag{27}$$

Now, we just need to work out the sign of this expression (27) in order to be able to sign equation (26).

The first thing we observe is that prudence is not enough to determine the sign of the effect of wealth on the scale of a project (contradicting Bonilla and Vergara, 2013). Notice that, in the same way as in the previous section, i.e. using the property of $E\widetilde{x}\widetilde{y} = E\widetilde{x}E\widetilde{y} + Cov(\widetilde{x}, \widetilde{y})$, equation (27) can be written as;

$$E_\theta u''\left(f(L^*,\theta) - wL^* + a\right) E_\theta\left(f_L(L^*,\theta) - w\right) + Cov\left(u'', f_L(L^*,\theta)\right) \tag{28}$$

The assumptions that are in place on f imply that as θ increases, $f_L(L^*,\theta)$ also increases. Similarly, as θ increases, $f(L^*,\theta)$ also increases, making $f(L^*,\theta) - wL^* + a$ increase. If we further assume prudence, $u''' > 0$, then u'' is an increasing function, and so we arrive at the conclusion that when θ increases, $u''\left(f(L^*,\theta) - wL^* + a\right)$ also increases. Thus, $Cov\left(u'', f_L(L^*,\theta)\right) > 0$. However, $E_\theta u''\left(f(L^*,\theta) - wL^* + a\right) < 0$ and $E_\theta\left(f_L(L^*,\theta) - w\right) > 0$, so it happens that $E_\theta u''\left(f(L^*,\theta) - wL^* + a\right) E_\theta\left(f_L(L^*,\theta) - w\right) < 0$.

Thus the sign of $E_\theta u''\left(f(L^*,\theta) - wL^* + a\right)\left(f_L(L^*,\theta) - w\right)$ is ambiguous under prudence, and therefore, to sign (26) we need more structure in our entrepreneurial model. Consequently, to put a sign on the effect, we need to use a different approach. It is convenient to attempt to link the sign of the effect to a characteristic of risk aversion, and that is what we do next.

Proposition 2. *When risk aversion is small enough, an increase in wealth for entrepreneurs induces larger sizes of the ventures ($L_a^*(a) > 0$), as long as the decision maker exhibits DARA preferences.*

Proof. Since the Arrow-Pratt absolute risk aversion is defined by

$$A(y) = -\frac{u''(y)}{u'(y)} \tag{29}$$

we can write

$$u''(y) = -A(y)u'(y) \tag{30}$$

Substitute (30) into the equation that we wish to sign (equation(27)) to get

$$\begin{aligned} & E_\theta u''\left(f(L^*,\theta) - wL^* + a\right)\left(f_L(L^*,\theta) - w\right) \\ = {} & E_\theta\left(-A(f(L^*,\theta) - wL^* + a)\right) u'\left(f(L^*,\theta) - wL^* + a\right)\left(f_L(L^*,\theta) - w\right) \\ = {} & E_\theta\left(-A(f(L^*,\theta) - wL^* + a)\right) k(L^*,\theta) \end{aligned} \tag{31}$$

As above, assume that $k(L^*,\theta)$ increases with θ, which as we have already seen from the previous section, defines an upper bound on risk aversion. We also need to place an assumption on risk aversion, $A(y)$, so we shall assume decreasing absolute risk aversion (DARA), that is, $-A(y)$ increases with y.

Now, proceed in exactly the same way as above, with $\overline{\theta} \leftarrow k(L^*,\overline{\theta}) = 0$. Since $-A(y)$ is an increasing function, and since $f(L^*,\theta) - wL^* + a$ increases with θ, it happens that

$$\int_{-\infty}^{\overline{\theta}} \left(-A(f(L^*,\theta) - wL^* + a)\right) g(\theta)d\theta \;<\; \int_{-\infty}^{\overline{\theta}} \left(-A(f(L^*,\overline{\theta}) - wL^* + a)\right) g(\theta)d\theta \tag{32}$$

$$\int_{\overline{\theta}}^{\infty} \left(-A(f(L^*,\theta) - wL^* + a)\right) g(\theta)d\theta \;>\; \int_{\overline{\theta}}^{\infty} \left(-A(f(L^*,\overline{\theta}) - wL^* + a)\right) g(\theta)d\theta \tag{33}$$

We already know that, since $k(L^*,\theta)$ increases with θ, we have that

$$\begin{aligned} k(L^*,\theta) &< 0 \text{ for all } \theta < \overline{\theta} \\ k(L^*,\theta) &> 0 \text{ for all } \theta > \overline{\theta} \end{aligned} \tag{34}$$

Thus, multiplying (32) and (33) by $k(L^*, \theta)$ we get

$$\begin{aligned}\int_{-\infty}^{\overline{\theta}} \left(-A(f(L^*, \theta) - wL^* + a)\right) k(L^*, \theta)g(\theta)d\theta &> \\ \int_{-\infty}^{\overline{\theta}} \left(-A(f(L^*, \overline{\theta}) - wL^* + a)\right) k(L^*, \theta)g(\theta)d\theta & \qquad (35)\end{aligned}$$

and

$$\begin{aligned}\int_{\overline{\theta}}^{\infty} \left(-A(f(L^*, \theta) - wL^* + a)\right) k(L^*, \theta)g(\theta)d\theta &> \\ \int_{\overline{\theta}}^{\infty} \left(-A(f(L^*, \overline{\theta}) - wL^* + a)\right) k(L^*, \theta)g(\theta)d\theta & \qquad (36)\end{aligned}$$

Now, summing up the two left-hand side and right-hand side together from (35) and (36) we get

$$\begin{aligned}\int_{-\infty}^{\infty} \left(-A(f(L^*, \theta) - wL^* + a)\right) k(L^*, \theta)g(\theta)d\theta &> \\ \int_{-\infty}^{\infty} \left(-A(f(L^*, \overline{\theta}) - wL^* + a)\right) k(L^*, \theta)g(\theta)d\theta & \qquad (37)\end{aligned}$$

And operating on the right-hand side of (37) we obtain

$$\begin{aligned}&\int_{-\infty}^{\infty} \left(-A(f(L^*, \overline{\theta}) - wL^* + a)\right) k(L^*, \theta)g(\theta)d\theta \\ &= -A(f(L^*, \overline{\theta}) - wL^* + a) \int_{-\infty}^{\infty} k(L^*, \theta)g(\theta)d\theta \\ &= -A(f(L^*, \overline{\theta}) - wL^* + a)E_\theta k(L^*, \theta) = 0 \qquad (38)\end{aligned}$$

The final equality comes from the fact that $E_\theta k(L^*, \theta) = 0$ from the first-order condition for the problem with $u(y)$. □

This result proves exactly what we are looking for, i.e. when risk aversion is small enough, an increase in wealth for entrepreneurs induces larger sizes of the ventures

$(L_a^*(a) > 0)$ when the decision maker exhibits DARA preferences. Interestingly, though, in contrast to much of the existing literature on this topic, DARA alone or (as shown above) prudence, are not sufficient to guarantee the result that wealthier entreprenuers embark upon larger business ventures. It is also necessary to have an upper bound on risk aversion itself.

4 The Transition Effect

To analyze the transition effect we will focus our attention into the cases in which small changes in the parameters of the model will induce a transition into entrepreneurship for the indifferent decision maker.

4.1 Increases in parameters w and a

An increase in w

Our main focus here is to understand what happen with the partition set of workers and entrepreneurs when there is an exogenous change in the wage w. It turns out that the response is very intuitive.

We know that the marginal entrepreneur is indifferent between working as an employee or going into entrepreneurship, that is;

$$E_\theta u\left(f\left(L^*(w,\overline{a}),\theta\right) - wL^*(w,\overline{a}) + \overline{a}\right) = u\left(w + \overline{a}\right) \tag{39}$$

Logic implies that an increase in w should make an inititally marginal agent strictly prefer becoming an employee over transitioning into entreprenuership. For that to happen, we would require that an increase in w causes a larger increase in the right-hand side of (39) than in its left-hand side. However, the derivative of the right-hand side is just $u'(w+\overline{a})$ which is strictly positive. The derivative of the left-hand side is;

$$\begin{aligned} &E_\theta u'\left(f\left(L^*(w,\overline{a}),\theta\right)-wL^*(w,\overline{a})+\overline{a}\right)\left[(f_L(L^*(w,\overline{a}),\theta)-w)L_w(w,\overline{a})-L^*(w,\overline{a})\right] \\ &\quad = E_\theta u'\left(f\left(L^*(w,\overline{a}),\theta\right)-wL^*(w,\overline{a})+\overline{a}\right)(f_L(L^*(w,\overline{a}),\theta)-w)L_w(w,\overline{a}) \\ &\qquad -E_\theta u'\left(f\left(L^*(w,\overline{a}),\theta\right)-wL^*(w,\overline{a})+\overline{a}\right)L^*(w,\overline{a}) \qquad (40) \end{aligned}$$

The first-order condition for an optimal entrepreneurial choice, equation (2), implies that the first term of this is 0, so we are left with

$$-E_\theta u'\left(f\left(L^*(w,\overline{a}),\theta\right)-wL^*(w,\overline{a})+\overline{a}\right)L^*(w,\overline{a})<0 \qquad (41)$$

Since the derivative of the right-hand side of the equation defining the marginal agent is positive and the derivative on the left-hand side is negative, we have the result that an increase in w causes an initially marginal agent to strictly prefer employment over entrepreneurship, as expected.

Intuitively, this means that an increase in the wage is a re-assigning of resources from the residual profits of the entrepreneur into the payment received by the employee, making paid employment with no risk more attractive for the (initially) indifferent decision maker.

An increase in a

The transition effect when there is a change in a implies that a decision maker with $a < \overline{a}$ decides to stay in secure employment, while another decision maker with $a > \overline{a}$ would transition into entrepreneurship. In this analysis we have to assume that the marginal transition wealth $\overline{a}$ is strictly positive and sufficiently small so that some agents transition and others do not and therefore, a partition of the set of decision makers is possible. Then, as we already know, the marginal or indifferent entrepreneur satisfies,

$$E_\theta u\left(f\left(L^*(w,\overline{a}),\theta\right) - wL^*(w,\overline{a}) + \overline{a}\right) = u\left(w + \overline{a}\right) \tag{42}$$

In order that a marginal increase in wealth induces the decision maker to become an entrepreneur we require that the derivative of the left-hand side of (42) in $\overline{a}$ be larger than the derivative of the right-hand side of (42) in $\overline{a}$, that is

$$E_\theta u'\left(f\left(L^*(w,\overline{a}),\theta\right) - wL^*(w,\overline{a}) + \overline{a}\right)\left(f_L\left(L^*(w,\overline{a}),\theta\right)L_a^*(w,\overline{a}) - wL_a^*(w,\overline{a}) + 1\right) > u'\left(w + \overline{a}\right) \tag{43}$$

then, the left-hand side of (43) equals

$$\begin{aligned} &E_\theta u'\left(f\left(L^*(w,\overline{a}),\theta\right) - wL^*(w,\overline{a}) + \overline{a}\right)\left(f_L\left(L^*(w,\overline{a}),\theta\right) - w\right)L_a^*(w,\overline{a}) \\ &+E_\theta u'\left(f\left(L^*(w,\overline{a}),\theta\right) - wL^*(w,\overline{a}) + \overline{a}\right) \end{aligned} \tag{44}$$

But since the first-order condition for the optimal investment by this investor is

$$E_\theta u'\left(f\left(L^*(w,\overline{a}),\theta\right) - wL^*(w,\overline{a}) + \overline{a}\right)\left(f_L\left(L^*(w,\overline{a}),\theta\right) - w\right) = 0 \tag{45}$$

Then, our required inequality is reduced to

$$E_\theta u'\left(f\left(L^*(w,\overline{a}),\theta\right)-wL^*(w,\overline{a})+\overline{a}\right)>u'\left(w+\overline{a}\right) \tag{46}$$

Now, if marginal utility is convex (i.e. if the decision maker is prudent), then by Jensen's inequality we know that

$$E_\theta u'\left(f\left(L^*(w,\overline{a}),\theta\right)-wL^*(w,\overline{a})+\overline{a}\right)>u'\left(E_\theta f\left(L^*(w,\overline{a}),\theta\right)-wL^*(w,\overline{a})+\overline{a}\right) \tag{47}$$

and from the defining equation for the marginal investor, we already know that

$$E_\theta f\left(L^*(w,\overline{a}),\theta\right)-wL^*(w,\overline{a})>w$$

But then, risk aversion (decreasing marginal utility) implies that

$$u'\left(E_\theta f\left(L^*(w,\overline{a}),\theta\right)-wL^*(w,\overline{a})+\overline{a}\right)<u'\left(w+\overline{a}\right) \tag{48}$$

so that $E_\theta u'\left(f\left(L^*(w,\overline{a}),\theta\right)-wL^*(w,\overline{a})+\overline{a}\right)$ and $u'\left(w+\overline{a}\right)$ cannot easily be compared. So again, refuting previous literature, the comparative static of the effect of wealth changes on the scale of projects cannot be compared using only prudence and risk aversion, and that meas that we need a different approach to understand the transition effect, and that approach will be based on the DARA property.

Proposition 3. *When absolute risk aversion coefficient is small enough and decision makers exhibit DARA preferences, an increase in the wealth level a guarantees the transition into entrepreneurship for the indifferent or marginal entrepreneur.*

Proof. Going back to the original equality for the marginal decision maker,

$$E_\theta u\left(f\left(L^*(w,\overline{a}),\theta\right)-wL^*(w,\overline{a})+\overline{a}\right)=u\left(w+\overline{a}\right)$$

We know that due to risk aversion, this equation implies $E_\theta f(L^*(w,\overline{a}),\theta) - wL^*(w,\overline{a}) > w$. Therefore, there exists a risk-premium $\pi(w,\overline{a})$ such that;

$$E_\theta u(f(L^*(w,\overline{a}),\theta) - wL^*(w,\overline{a}) + \overline{a}) = u(E_\theta f(L^*(w,\overline{a}),\theta) - wL^*(w,\overline{a}) + \overline{a} - \pi(w,\overline{a})) \tag{49}$$

where $\pi(w,a)$ is the risk-premium function. We can now write the utility equivalence of the marginal decision maker as

$$u(E_\theta f(L^*(w,\overline{a}),\theta) - wL^*(w,\overline{a}) + \overline{a} - \pi(w,\overline{a})) = u(w - \overline{a}) \tag{50}$$

Again, we would like to show that the derivative in $\overline{a}$ of the left-hand side is greater than the derivative of the right-hand side;

$$u'(Ef(L^*(w,\overline{a}),\theta) - wL^*(w,\overline{a}) + \overline{a} - \pi(w,\overline{a}))(Ef_L(L^*(w,\overline{a}),\theta) - w)L_a^*(w,\overline{a}) + 1 - \pi_a(w,\overline{a})) > u'(w+\overline{a}) \tag{51}$$

But, due to the utility equivalence with the risk premium, we know that

$$Ef(L^*(w,\overline{a}),\theta) - wL^*(w,\overline{a}) + \overline{a} - \pi(w,\overline{a}) = w + \overline{a} \tag{52}$$

and consequently

$$u'(Ef(L^*(w,\overline{a}),\theta) - wL^*(w,\overline{a}) + \overline{a} - \pi(w,\overline{a})) = u'(w+\overline{a}) \tag{53}$$

Then, plugging (53) into (51) and dividing both sides by $u'(w+\overline{a})$, we can see that what is required translates into

$$(Ef_L(L^*(w,\overline{a}),\theta) - w)L_a^*(w,\overline{a}) + 1 - \pi'(w,\overline{a}) \ > \ 1$$

$$(Ef_L(L^*(w,\overline{a}),\theta) - w)L_a^*(\overline{a}) \ > \ \pi_a(w,\overline{a}) \tag{54}$$

Under DARA, we know that the risk-premium is a decreasing function of wealth, i.e., $\pi_a(w, \overline{a}) < 0$. We have already shown above that for any risk-averse investor $Ef_L\left(L^*(a), \theta\right) - w > 0$. Therefore, subject to DARA and risk aversion being limited from above by the expression of the left-hand side of (25) we can be sure that $L_a^*(a) > 0$. □

Thus, the same conditions that are required for the optimal scale to increase with wealth imply that the marginal investor will transition into entrepreneurship if his wealth increases. That is, the transition effect can be guaranteed under the same conditions as the scale effect; we require DARA and that risk aversion to be sufficiently low.

4.2 Transition and risk aversion

In this section we will study the effect of an increase in risk aversion for the indifferent decision maker. We will now ammend out notation slightly to simplify our results. We can identify decision makers by a pair (u, a), where u is the utility function, and a is initial (risk-free) wealth. A given individual i then, with (u_i, a_i), needs to decide whether to become an employee, and have final wealth of $w + a$ and utility level of $u(w + a)$, or to become an entrepreneur, and have stochastic final wealth of that we will denote by

$$f(L_i^*(a_i), \theta) - L_i^*(a_i)w + a_i = \widetilde{x}_i^*(a_i) + a_i \tag{55}$$

where $L_i^*(a_i)$ is the optimal labor choice of individual i if he becomes an entrepreneur. In all that follows, it is understood that expectations are taken with respect to θ, so instead of writing $E_\theta h(\theta)$ for the expectation of any function h, we will simply write Eh.

Assume that individual i is indifferent between being an employee and being an entrepreneur;

$$Eu_i\left(\widetilde{x}_i^*(a_i) + a_i\right) = u_i(w + a_i) \tag{56}$$

Now consider the decision of a second individual, say individual j, who is identified by (u_j, a_j). Assume that u_j is more risk-averse than u_i, and that $a_j = a_i = a$. Due to individual j being more risk-averse, we know that there exists a function, $H(u)$, which is increasing and strictly concave, such that $u_j(z) = H(u_i(z))$. We also know that individual j will have a different optimal labor choice than will individual i, that is, if $\succ_j$ stands for the preference relation over lotteries for individual j, then for that individual we have $\widetilde{x}_j^*(a) + a \succ_j \widetilde{x}_i^*(a) + a$, and vice-versa for individual i.

By the definition above of j being more risk averse than i, and using the fact that H is strictly concave and Jensen's inequality, we get:

$$Eu_j(\widetilde{x}_j^* + a) = EH(u_i(\widetilde{x}_j^* + a)) < H\left(Eu_i(\widetilde{x}_j^* + a)\right) \tag{57}$$

But due to individual i having a strict preference for $\widetilde{x}_i^* + a$ over $\widetilde{x}_j^* + a$, we also know that $Eu_i(\widetilde{x}_j^* + a) < Eu_i(\widetilde{x}_i^* + a)$, which (due to the fact that H is an increasing function) implies

$$H\left(Eu_i(\widetilde{x}_j^* + a)\right) < H\left(Eu_i(\widetilde{x}_i^* + a)\right) \tag{58}$$

Thus we know that

$$Eu_j(\widetilde{x}_j^* + a) < H\left(Eu_i(\widetilde{x}_i^* + a)\right) \tag{59}$$

Finally, since individual i is indifferent between entrepreneurship and employment, $Eu_i(\widetilde{x}_i^* + a) = u_i(w + a)$. Substituting this into the previous inequality, we can write it as

$$Eu_j(\widetilde{x}_j^* + a) < H\left(u_i(w + a)\right) \tag{60}$$

And, by definition, $H\left(u_i(w + a)\right) = u_j(w + a)$, so we end up with

$$Eu_j(\widetilde{x}_j^* + a) < u_j(w + a) \tag{61}$$

That is:

Proposition 4. *If a given individual is indifferent between employment and entrepreneurship, then any other individual with the same level of wealth and a more risk-averse utility function will strictly prefer employment over entrepreneurship.*

Lemma 2. *If a given individual is indifferent between employment and entrepreneurship, then any other individual with the same level of wealth and a less risk-averse utility function will strictly prefer entrepreneurship over employment.*

Proof. An individual k is less risk averse than i if $u_i(z) = H(u_k(z))$ with H strictly increasing and concave. Therefore, $u_k(z) = H^{-1}(u_i(z))$ where H^{-1} is strictly increasing and convex. Using essentially the same argument as above, we have

$$Eu_k(\widetilde{x}_k^* + a) > Eu_k(\widetilde{x}_i^* + a) = EH^{-1}(u_i(\widetilde{x}_i^* + a)) > H^{-1}(Eu_i(\widetilde{x}_i^* + a)) = H^{-1}(u_i(w + a)) = u_k(w + a) \tag{62}$$

□

This guarantees that any decision maker who is less risk averse than the indifferent entrepreneur would prefer to transition into entrepreneurship instead of being an employee.

5 Stochastic dominance

The final comparative statics effects that we present here are when the underlying density corresponding to the choice of being an entrepreneur undergoes a stochastic dominant shift. Here, we restrict our attention to shifts of either first or second order only.

5.1 Stochastic dominance and the transition effect

Intuitively, a positive stochastic dominant shift in the density will lead to a greater propensity to switch from employment to entrepreneurship.

Proposition 5. *A first or a second-order stochastic dominant shift in the distribution of results induces the indifferent decision maker to transition into risk entrepreneurship.*

Proof. Assume that initially the density is given by some probability density $g_1(\theta)$, and that given that density, an entrepreneur with wealth a establishes a firm of size

$L_1^*(a)$. Then, if that individual is indifferent between employment and entrepreneurship, we would have

$$E_1 u\left(f(L_1^*(a),\theta) - L_1^*(a)w + a\right) = u(w+a) \tag{63}$$

where the expectation is taken with respect to $g_1(\theta)$, as is indicated by writing the expectation E_1. Now, our assumptions on utility are that it is strictly increasing and strictly concave, thus if the density changes to another one, say $g_2(\theta)$, that dominates density 1 under either first or second order stochastic dominance, then it must occur that

$$E_2 u\left(f(L_1^*(a),\theta) - L_1^*(a)w + a\right) > E_1 u\left(f(L_1^*(a),\theta) - L_1^*(a)w + a\right) \tag{64}$$

For equation (64) to be true, we only need that for first-order stochastic dominant shifts the utility function exhibits positive marginal utility, while for second-order stochastic dominant shifts we need that the utility function also exhibits risk aversion. Both assumptions are part of the initial assumptions of the model and therefore, if we allow the individual to choose the optimal scale of the firm under the new density we get;

$$E_2 u\left(f(L_2^*(a),\theta) - L_2^*(a)w + a\right) > E_2 u\left(f(L_1^*(a),\theta) - L_1^*(a)w + a\right) \tag{65}$$

Finally, substituting back into the original indifference equation, we now know that

$$E_2 u\left(f(L_2^*(a),\theta) - L_2^*(a)w + a\right) > u(w+a) \tag{66}$$

that is, a stochastic dominant improvement in the density (of either first or second order), causes previously indifferent individuals to strictly prefer to become entrepreneurs. □

5.2 Stochastic dominance and the scale effect

Now assume that an individual has already transitioned into entrepreneurship, and we ask how a stochastic dominant shift in the density (again, of either first or second order), affects the optimal size of the business venture (as measured by the employment of labour). To study this effect, we need only consider the first-order condition for the optimal scale:

$$Eu'\left(f(L^*(a),\theta) - L^*(a)w + a\right)\left(f'_L(L^*(a) - w\right) = 0 \tag{67}$$

We can write this as the expectation of a given function equal to zero:

$$Ek(L^*(a),\theta) = 0 \tag{68}$$

where of course $k(L^*(a),\theta) = u'\left(f(L^*(a),\theta) - L^*(a)w + a\right)\left(f'_L(L^*(a) - w\right)$. We have assumed already that $k(L^*(a),\theta)$ increases with θ, which implies a maximal value for absolute risk aversion. And it can also easily be seen that $k(L,\theta)$ decreases with L (which is the second-order equation for the optimal scale problem).

Under a first-order stochastic dominant shift in the density, since $k(L^*(a),\theta)$ increases with θ, it happens that the expected value of $k(L^*(a),\theta)$ must increase. Therefore, a first-order stochastic dominant shift in the density will cause the expected value of $k(L^*(a),\theta)$ to become strictly positive, rather than 0 as would be required for an optimal scale. The individual will adjust L to bring the expected value of $k(L^*(a),\theta)$ back to 0, and since $k(L^*(a),\theta)$ decreases with L, this is achieved by increasing L^*. Therefore;

Proposition 6. *A first-order stochastic dominant shift in the density will increase the optimal scale of the enterprise, subject to our condition that absolute risk aversion is sufficiently low.*

The intuition for a first-order stochastic dominant shift in the density carries over to shifts of the second order, but now the condition for the optimal scale to increase is that $k(L, \theta)$ needs to be both increasing and concave in θ. Of course, one can make the relevant assumptions on the utility function u and the production function f such that $k(L, \theta)$ is indeed concave in θ, which will be what is required for a second-order stochastic dominant shift in the density to increase the optimal scale. Specifically, we have;

$$\frac{\partial k}{\partial \theta} = u''(\cdot) f_\theta(L, \theta)(f_L(L, \theta) - w) > 0 \tag{69}$$

$$\frac{\partial^2 k}{\partial \theta^2} = u'''(\cdot) f_\theta(L, \theta)^2 (f_L(L, \theta) - w) + u''(\cdot) f_{\theta\theta}(L, \theta)(f_L(L, \theta) - w) + u''(\cdot) f_\theta(L, \theta) f_{L\theta}(L, \theta) \tag{70}$$

Under our initial assumptions of prudence, $u''' > 0$, $f_\theta(L, \theta) > 0$, and $f_{L\theta}(L, \theta) < 0$, and since at the optimal scale (under strict risk aversion) we have $(f_L(L, \theta) - w) > 0$, then the first term of $\frac{\partial^2 k}{\partial \theta^2}$ is strictly positive, the second term has the opposite sign to $f_{\theta\theta}(L, \theta)$, and the third term is again positive. Thus, a necessary, but not sufficient, condition for $k(L, \theta)$ to be concave in θ is that $f_{\theta\theta}(L, \theta) > 0$. The sufficient condition is:

$$u'''(\cdot) f_\theta(L, \theta)^2 (f_L(L, \theta) - w) + u''(\cdot) \left(f_{\theta\theta}(L, \theta)(f_L(L, \theta) - w) + f_\theta(L, \theta) f_{L\theta}(L, \theta) \right) < 0 \tag{71}$$

which implies that

$$u''(\cdot)\left(f_{\theta\theta}(L,\theta)(f_L(L,\theta)-w)+f_\theta(L,\theta)f_{L\theta}(L,\theta)\right) \;<\; -u'''(\cdot)f_\theta(L,\theta)^2(f_L(L,\theta)-w) \tag{72}$$

and this translates into our final condition

$$\frac{f_{\theta\theta}(L,\theta)(f_L(L,\theta)-w)+f_\theta(L,\theta)f_{L\theta}(L,\theta)}{f_\theta(L,\theta)^2(f_L(L,\theta)-w)} \;>\; -\frac{u'''(\cdot)}{u''(\cdot)} \tag{73}$$

Thus, the Arrow-Pratt measure of absolute prudence needs to be sufficiently low, but with reference to a much more complex upper bound;

Proposition 7. *A second-order stochastic dominant shift in the density will increase the optimal scale of the enterprise under the assumptions of absolute risk aversion being sufficiently low, and absolute prudence being sufficiently low.*

6 Concluding Remarks

The existing literature on the choices of economic agents regarding entrepreneurship tends to assume that risk aversion, decreasing risk aversion, and prudence are sufficient to guarantee all reasonable and intuitive results. However, we have shown that certain of the key results are also dependent upon risk aversion being sufficiently low, and (for stochastic dominance and the scale effect) prudence being sufficiently low. That is, risk aversion, DARA and prudence alone cannot guarantee the often assumed comparative statics related to entreprenuerial choice.

Concretely, we have shown that the sufficient condition for a risk averse agent to transition from employment to entreprenuership when his wealth increases also requires that the Arrow-Pratt absolute risk aversion coefficient to be smaller than an upper bound. Likewise, our results also show that the same condition on risk aversion is required to guarantee that an increase in risk free wealth will generate a larger scale of business for any agent currently acting as an entrepreneur, and for an increase in risk aversion (with constant wealth) to decrease the optimal scale of the business. The effects generated by stochastic dominance for the optimal scale of the entrepreneur's business are more complex, and (for second-order stochastic dominance) also require an upper bound on prudence.

Our results suggest that the two bounds on utility are important to study. They both imply complex functions of the characteristics of the underlying density corresponding to the business venture. We prefer to leave this analysis on our research agenda for now.

References

[1] Ahn, T. 2009. "Attitudes toward risk and self-employment of young workers". *Labour Economics*, 17, pp. 434-442.

[2] Bonilla C. and M. Vergara. 2013. "Credit Rationing or Entrepreneurial Risk Aversion? A Comment" *Economics Letters*, 120(2), pp. 329 - 331.

[3] Caliendo, M., F. Fossen and A. Kritikos. 2010. "The impact of risk attitudes on entrepreneurial survival". *Journal of Economic Behavior & Organization*, 76(1), pp. 45-63.

[4] Cramer, J., J. Hartog, N. Jonker and C. Van Praag. 2002. "Low risk aversion encourages the choice for entrepreneurship: An empirical test of truism". *Journal of Economic Behavior & Organization*, 48(1), pp. 29-36.

[5] Cressy, R. 2000. "Credit rationing or entrepreneurial risk aversion? An alternative explanation for the Evans and Jovanovic finding." *Economics Letters*, 66(2), pp. 235-240.

[6] Eeckhoudt, L. and R. Huang. 2012. "Precautionary effort: A new look". *Journal of Risk and Insurance*, 79(2), pp. 585 - 590

[7] Eeckhoudt, L., Fiori, A. and Gianin, E. (2016). "Loss-averse preferences and portfolio choices: An extension ". *European Journal of Operational Research*, 249: pp. 224-230.

[8] Evan, D and B. Jovanovic. 1989. "An estimated model of entrepreneurial under liquidity constraint." *Journal of Political Economy*, 97(4), pp. 808-827.

[9] Hartog, J., A. Ferrer-i-Carbonell and N. Jonker. 2002. "Linking measured risk Aversion to individual characteristics". *Kyklos*, 55, pp. 3-26.

[10] Hvide, H. K., G. A. Panos. 2014. "Risk tolerance and entrepreneurship". *Journal of Financial Economics*, 111, pp. 200-223.

[11] Kan, K., W., Tsai. 2006. "Entrepreneurship and risk aversion". *Small Business Economics*, 26, pp. 465-474.

[12] Kanbur, S., M. 1979. "Of risk taking and the personal distribution of Income." *Journal of Political Economics* 87, pp. 760-797.

[13] Kanbur, S. M. 1981. "Risk taking and taxation." *Journal of Political Economics* 15, pp. 163-184.

[14] Kihlstrom, R. and J. Laffont. 1979. "A general equilibrium entrepreneurial theory of new firm formation based on risk aversion." *Journal of Political Economy* 87, pp. 304-316.

[15] Kimball, M. S. 1990. "Precautionary saving in the small and in the Large". *Econometrica*, 58(1), pp. 53-73.

[16] Knight, F. 1921. "Risk, Uncertainty, and Profit." Hart, Schaffner, and Marx Prize Essays, no. 31. Boston and New York: Houghton Mifflin.

[17] Koudstaal, M., R. Sloof, and M. van Praag. "Risk, Uncertainty, and Entrepreneurship: Evidence from a Lab-in-the-Field Experiment". *Management Science*, 62(10), pp. 2897 – 2915.

[18] Menegatti, M.(2014). "New results on the relation among risk aversion, prudence and temperance ". *European Journal of Operational Research*, 232: pp. 613-617.

[19] Vereshchagina, G. and H. Hopenhayn. 2009. "Risk taking by entrepreneurs". *The American Economic Review*, 99(5), pp. 1808-1830.

[20] van Praag, C. W. and J.S. Cramer. 2001. "The roots of entrepreneurship and labour demand: Individual ability and low risk aversion". *Economica*, 68 pp. 45-62.

[21] Wang, J. and J. li. 2014. "Precautionary effort: nother trait for prudence". *Journal of Risk and Insurance*, 82(4), pp. 977 - 983