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On Estimating Long-Run Effects In Models with Lagged Dependent Variables

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On Estimating Long-Run Effects In Models with Lagged Dependent Variables

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Abstract: A common procedure in economics is to estimate long-run effects from models with lagged dependent variables. For example, macro panel studies frequently are concerned with estimating the long-run impacts of fiscal policy, international aid, or foreign investment. This note points out the hazards of this practice. We use Monte Carlo experiments to demonstrate that estimating long-run impacts from dynamic models produces unreliable results. Biases can be substantial, sample ranges very wide, and hypothesis tests can be rendered useless in realistic data environments. There are three reasons for this poor performance. First, OLS estimates of the coefficient of a lagged dependent variable are downwardly biased in finite samples. Second, small biases in the estimate of the lagged, dependent variable coefficient are magnified in the calculation of long-run effects. And third, and perhaps most importantly, the statistical distribution associated with estimates of the *LRP* is complicated, heavy-tailed, and difficult to use for hypothesis testing.

Keywords: Hurwicz bias, Auto-Regressive Distributed-Lag (ARDL) models, Dynamic Panel Data (DPD) models, DPD estimators, long-run impact, long-run propensity, Fieller's method, indirect inference, jackknifing

JEL Classifications: C22, C23

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I. INTRODUCTION

This note points out the hazards of estimating long-run effects from models with lagged dependent variables. We illustrate the problem in the context of the following ARDL(*p*,*q*) model where *p=1* and *q=0*:

(1) $y_t = \beta_0 + \beta_x x_t + \beta_y y_{t-1} + u_t, t = 1,2,\ldots,T.$

We assume $|\beta_y| < 1$. Within this framework, it is common to estimate the long-run effect of *x* on *y*, also known as the long-run propensity (*LRP*) of *x*, by

(2) $LRP = \hat{\beta}_x / (1 - \hat{\beta}_y),$

where $\hat{\beta}_x$ and $\hat{\beta}_y$ are the OLS estimates from Equation (1).[1] Another approach to estimating *LRP* is to rewrite Equation (1) in growth equation form as

(3.a) $\Delta y_t = \beta_0 + \delta(y_{t-1} - \theta x_t) + u_t$, or

(3.b) $\Delta y_t = \beta_0 + \delta(y_{t-1} - \theta x_{t-1}) + \beta_x \Delta x_t + u_t,$

where $\delta = (\beta_y - 1)$, and the value of *LRP* is estimated by $\hat{\theta}$ via maximum likelihood.

Examples of both approaches abound. It is common to see the former approach presented in popular econometrics textbooks such as Judge et al. (1988, page 737), Johnston & Dinardo (1997, page 245), Hill et al. (2001, page 328), Kennedy (2008, page 322), Gujarati and Porter (2010, page 378), Greene (2012, page 423), and Wooldridge (2012, page 635). Recent journal articles employing this approach include Baltagi et al. (2009), Hoque and Yusop (2010); Heid et al. (2012); Islam et al. (2013), and Li et al. (2015). The latter approach is common in the macro panel literature. Recent examples here include Gemmell et al. (2011), Ojede and Yamarik (2012), Klomp and De Hann (2013), Calderón et al. (2015), and Eberhardt and Presbitero (2015), to just name a few.

[1] Generalization to higher orders of *p* and *q* is straightforward.

Unfortunately, this procedure has serious pitfalls. It is well known that OLS estimation of autoregressive (AR) models -- while consistent -- is biased in finite samples (Hurwicz, 1950; Phillips, 1977). It follows that estimates of the *LRP* will likewise be biased. Even worse, the *LRP* is a nonlinear function of the AR coefficient(s), and a small bias in the denominator can be greatly magnified, especially when the AR coefficients are close to the unit circle.

Marriott and Pope (1954) illustrate this in the case when $p = 1$. The first-order bias for OLS estimation of β_y is $-(1 + 3\beta_y)/T$. When $\beta_y = 0.95$ and $T = 100$, this implies a bias of -0.0385, or 4% of the true value. However, the associated bias in the *LRP* is 43.5%, since the corresponding estimate of *LRP* is 11.3, compared to its true value of 20. Things can get even worse when higher order AR terms are included in the equation. As pointed out by Patterson (2000), small biases in the estimates of the individual AR coefficients accumulate, often in a reinforcing rather than offsetting manner.

And there is yet another issue. The *LRP* of Equation (2) is the ratio of two normally distributed random variables. As is well known, when both random variables are independent and have zero means, their ratio is distributed Cauchy. However, when the two variables have non-zero means, Hinkley (1969) has shown that the associated distribution is quite complicated. The shape of its density can be unimodal, bimodal, symmetric, or asymmetric depending on the value of the coefficient of the denominator variable. Due to this fact, it is difficult to work with the *LRP* and develop appropriate statistical tests.

At some level, most of the above is already known. What is not known – as evidenced by the widespread use of these procedures – is the practical importance of these problems for estimates of long-run effects in realistic data environments. That is the main contribution of this study.

We proceed as follows. Section 2 illustrates the problem by demonstrating the finite sample bias associated with estimating *LRP* from univariate time series data. It explores a number of solutions that have been suggested in the literature, specifically (i) Fieller's method for testing hypotheses about ratios of parameters, and (ii) jackknifing and indirect inference for correcting biases in coefficients estimated from an ARDL model. Section 3 extends the analysis in two directions. It first considers the estimation of *LRP* in a panel data setting, investigating the performance of several popular dynamic panel data estimators (Dynamic Fixed Effects, Anderson-Hsaio, Difference GMM, and System GMM). It also explores the effect of nonstationarity, and the use of Dynamic OLS in both a univariate time series and panel data setting. Section 4 then compares a wide variety of panel data estimators to identify which estimators perform best. In addition to the estimators already mentioned, it studies Mean Group, Common Correlated Effect Mean Group, and Augmented Mean Group estimators. Relatedly, it investigates the widely adopted practice of transforming annual data into 5-year averages. Section 5 summarizes the main findings of our analysis and draws implications for the estimation of long-run effects.

2. ESTIMATION OF *LRP* USING UNIVARIATE TIME SERIES

Hurwicz bias. We introduce the problem of finite sample bias by using Monte Carlo experiments to study OLS estimation of the model:

(4) $y_t = \beta_0 + \beta_y y_{t-1} + u_t,\ u_t \sim NID(0,1),\ t = 1,2,...,T.$

The error terms u_t are generated independently of the lagged dependent variable values, y_{t-1}. T takes values between 10 and 1000 ($T = 10, 30, 50, 100,$ and 1000), $\beta_0 = 0$, and β_y takes values between 0.60 and 0.95 ($\beta_y = 0.60, 0.70, 0.80, 0.90,$ and 0.95). For each simulated sample, OLS is used to estimate the value of β_y.[2] 10,000 replications were run for each of the (T, β_y) combinations.

[2] While the data generating process (DGP) in these, and all subsequent, experiments set the value of the constant term to zero, the estimated models include an intercept.

The results of these Monte Carlo experiments are reported in TABLE 1.[3] The top panel of the table reports the average estimated value of $\hat{\beta}_y$ across the 10,000 replications. The bottom panel reports Type I error rates, where the values are the rejection rates of the null hypothesis, H_0: $\beta_y = its\ true\ value$. For both panels, the columns represent the different sample sizes, and the rows represent the different true values of β_y.

For example, when $\beta_y = 0.80$ and *T=50*, the average estimated value of β_y = 0.73. When *T=1000,* the average estimated value of β_y increases to 0.80, illustrating the consistency of the OLS estimator. Likewise, when $\beta_y = 0.80$ and *T=50*, the rejection rate of the null hypothesis H_0: $\beta_y = its\ true\ value$ -- given a significance level of 5% -- is 0.08. This rate declines as the sample size gets larger, falling to 0.05 when *T=1000*. The finite sample bias from estimating Equation (4) is well known as Hurwicz bias (Hurwicz, 1950). While the biases may appear relatively small, as noted above, small biases in β_y can result in large biases in *LRP*.

Estimation of long-run effects. The next set of Monte Carlo experiments add an explanatory variable to the data generating process (DGP) of Equation (4):

(5) $$y_t = \beta_0 + \beta_x x_t + \beta_y y_{t-1} + u_t,\ \ x_t, u_t \sim NID(0,1),\ \ t = 1,2,\ldots,T;$$

where $\beta_0 = 0$, $\beta_x = 1$, β_y takes values 0.60, 0.70, 0.80, 0.90, and 0.95 (same as above), and $\mathrm{Cov}(x_t, u_t) = 0$. The corresponding true *LRP* values are 2.5, 3.3, 5, 10, and 20.

TABLE 2 reports the results of these Monte Carlo experiments. The table is organized similarly to TABLE 1 with some minor differences. We now report three panels. The top panel reports the *median* (not average) value of the estimated *LRP* values across the 10,000

[3] Stata, Version 14 was used to produce TABLES 1-3 and 6-9. Matlab was used to produce TABLES 4 and 5. All the programs are available online at Dataverse: https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/SCFMVN .

replications.[4] The middle panel reports the 90 percent empirical sample range calculated by taking the 5th and 95th percentile values of the sorted 10,000 estimated *LRP* values. The bottom panel reports Type I error rates associated with the null hypothesis, H_0: $LRP = its\ true\ value$. We report simulation results for sample sizes of *T = 10, 50*, and *1000*.

For a given sample size, the bias in estimating *LRP* increases (in both absolute and percentage terms) as β_y approaches one. For example, when $\beta_y = 0.60$ and *T = 50*, the median estimated value of LRP is 2.4, compared to its true value of 2.5. When β_y increases to 0.90, corresponding to a higher *LRP* value of 10, the associated median estimate of *LRP* is 7.5, a bias of negative 25 percent. In addition, the 90 percent empirical sample range becomes substantially wider, and the Type I error rate deviates further from 0.05.

TABLE 2 illustrates two additional points. First, the bias associated with estimating *LRP* is a finite sample problem. As sample sizes increase, *LRP* values are estimated with greater precision and the sizes of the respective hypothesis tests converge to their true values. Second, the bias can be quite large for sample sizes routinely used by researchers. This will be further illustrated when we move our experiments into a panel data setting.

Using Fieller's method to test hypotheses about *LRP*. An alternative way to test *LRP* is based on Fieller's Theorem (1954). The original theorem proposes a procedure to obtain a confidence set for a ratio of two means of normal variables. Zerbe (1978) extends the theorem to ratios of two normal variables. Fieller's method has received limited attention in econometrics. An exception is Bolduc et al. (2010), who provide a recent economic application in the context of discrete choice models. Fieller's method has not, to the best of our knowledge, been applied to testing hypotheses about *LRP*. To apply Fieller's method to testing $\widehat{LRP} = \gamma$, we calculate the following test statistic,

[4] We report the median rather than the mean, because estimates of β_y very close to 1 can cause the *LRP* to be extremely large, making the sample mean uninformative.

(6) $$t(\gamma) = \frac{\hat{\beta}_x - \gamma(1-\hat{\beta}_y)}{(\gamma^2 \hat{v}_y + 2\gamma \hat{v}_{xy} + \hat{v}_x)^{0.5}},$$

which follows $N(0,1)$. In the test statistic $t(\gamma)$, $\hat{v}_x$ is the estimated variance of $\hat{\beta}_x$, $\hat{v}_y$ is the estimated variance of $\hat{\beta}_y$, and $\hat{v}_{xy}$ is the estimated covariance of $\hat{\beta}_x$ and $\hat{\beta}_y$.

TABLE 3 reports the results of applying Fieller's method to the Monte Carlo experiments in TABLE 2. The table consists of two panels. The top panel reproduces the Type I error rates from TABLE 2. The bottom panel reports the rejection rates when applying Fieller's method. While Fieller's method does not eliminate size distortions associated with testing hypotheses about *LRP*, it does offer improvement, sometimes substantial improvement. Unfortunately, we will find that Fieller's method does not generally improve hypothesis testing when using panel data.

Jackknifing and indirect inference. Jackknifing (JK) and indirect inference (II) are two methods that have been suggested for correcting the bias associated with estimating AR/ARDL models in finite samples (cf., Gourieroux et al., 2010; Dhaene and Jochmans, 2015). In this section, we apply these methods in order to obtain less biased estimates of β_y. We then investigate whether this improves the corresponding *LRP* estimates.

The jackknife estimator removes bias by combining maximum-likelihood estimates across subsamples. Let $\hat{\beta}$ be the maximum-likelihood based estimator of β using a full sample of observations ordered by time. Let this sample be given by $S = \{1, \dots, T\}$. We create two subsamples, $S_1 = \{1, \dots, \frac{T}{2}\}$ and $S_2 = \{\frac{T}{2} + 1, \dots, T\}$, and estimate β separately on each subsample. Denote these estimates $\hat{\beta}_{S_1}$ and $\hat{\beta}_{S_2}$. The corresponding jackknife estimator is $\hat{\beta}_{JK} = 2\hat{\beta} - 0.5(\hat{\beta}_{S_1} + \hat{\beta}_{S_2})$. $\hat{\beta}_{JK}$ will have smaller bias than the original estimator, $\hat{\beta}$; but -- as is apparent from its mathematical expression -- it will have larger variance.

Indirect inference is a simulation-based method for bias reduction. Suppose the true DGP takes the form $y_t = F(y_{t-1}, x_t|\boldsymbol{\theta})$. In our case, $\boldsymbol{\theta}$ is a vector $(\boldsymbol{\beta}, \sigma)$, where $\boldsymbol{\beta}$ consists of the regression parameters $(\beta_0, \beta_x, \beta_y)$ and σ is the standard error of the regression error term. Suppose that $\widehat{\boldsymbol{\theta}} = (\widehat{\boldsymbol{\beta}}, \hat{\sigma})$ is an estimator of $\boldsymbol{\theta}$ based on observed data. The II procedure is carried out in three steps. Step 1 generates a sequence of random errors of length *T* from the *F* distribution conditional on the estimated parameter $\widehat{\boldsymbol{\theta}}$. Step 2 chooses values for the parameter vector $\boldsymbol{\beta}$ using $\{x_t\}_{t=1}^T$ and the simulated random errors in Step 1 to generate the sequence $\{\tilde{y}_t(\boldsymbol{\beta})\}_{t=1}^T$. The $\{\tilde{y}_t(\boldsymbol{\beta})\}_{t=1}^T$ and $\{x_t\}_{t=1}^T$ values are then used to calculate an estimate of $\boldsymbol{\beta}$, $\delta(\boldsymbol{\beta})$. Step 3 repeats Steps 1 and 2 *M* times (*M* simulations) for a given set of $\boldsymbol{\beta}$ values, producing the estimates $\{\delta_m(\boldsymbol{\beta})\}_{m=1}^M$. The bias-reduced estimator is obtained by $\widehat{\boldsymbol{\beta}}_{II} = argmin_\beta(\widehat{\boldsymbol{\beta}} - \frac{1}{M}\sum_{m=1}^M \delta_m(\boldsymbol{\beta})\})$, where $\widehat{\boldsymbol{\beta}}$ is the parameter estimate based on observed data. The central idea of indirect inference is to choose $\boldsymbol{\beta}$ so that $\delta(\boldsymbol{\beta})$ and $\widehat{\boldsymbol{\beta}}$ are as close as possible.

We first demonstrate that jackknifing and indirect inference are effective at reducing the bias associated with estimating β_y in Equation (4). The top panel of TABLE 4 reports the OLS estimates of β_y. The next two panels report the JK and II estimates. As before, the Monte Carlo experiments have β_y take values 0.60, 0.70, 0.80, 0.90, and 0.95. Sample sizes range from *T=10* to *T=1000*.

As was foreshadowed by TABLE 1, OLS estimates of β_y in the ARDL model are downwardly biased in finite samples. For example, when $\beta_y = 0.80$ and *T=10*, the average OLS estimate of β_y is 0.62. In contrast, the corresponding, average JK and II estimates are 0.76 and 0.78. That these latter two estimates are close to the true value of β_y even for *T=10* is testimony to the effectiveness of these procedures to reduce Hurwicz bias. In fact, the JK and II estimates of β_y dominate the OLS estimates on the dimension of bias across the full range of β_y and *T* values. Unfortunately, these improvements do not translate into better estimates of *LRP*.

TABLE 5 reports 90 percent empirical sample ranges for the JK and II estimators and compares them those for OLS. The top panel reproduces the OLS intervals from TABLE 2. The next two panels report the JK and II estimates. Generally speaking, the ranges are wider, often substantially wider, for the JK and II estimates. For example, when $\beta_y = 0.80$ and *T=50*, OLS estimates of the ARDL model produce *LRP* values that range from 2.4 to 8.1, encompassing the true value of 5. This compares with 2.3 to 15.9, and 2.7 to 12.4 for the JK and II estimates, respectively.

This poorer performance of the JK and II estimators occurs despite the fact that these estimators generally produce median *LRP* values that are closer to their true values (not reported). However, the greater variance of these estimators generally dominates the smaller bias, so that the sample ranges are wider, and the associated estimates less efficient.

3. EXTENDING THE ANALYSIS TO PANEL DATA AND NONSTATIONARITY

<u>Panel data</u>. In this section, we extend the analysis to more practical situations by studying the problem in a panel data setting. The DGP for our experiments is given by:

(7) $$y_{it} = \beta_0 + \beta_x x_{it} + \beta_y y_{i,t-1} + \varepsilon_{it},\ i = 1,2,\ldots,N,\ t = 1,2,\ldots,T,$$

where *N=50, T=10,* $\beta_0 = 0$, $\beta_x = 1$, and β_y again takes values 0.60, 0.70, 0.80, 0.90, and 0.95. The error term, ε_{it}, is comprised of a fixed effect and a classical error term, $\varepsilon_{it} = a_i + u_{it}$, where each of these components are independently and identically standard normally distributed, $a_i, u_{it} \sim NID(0,1)$. The fixed effects are constructed to be uncorrelated with x_{it}, so there is no associated endogeneity. We do this to focus attention on the problem of estimating *LRP*, without introducing additional bias issues.

Given the values of β_y, the true *LRP* values are, again, 2.5, 3.3, 5, 10, and 20, respectively. We estimate the model using four dynamic panel data (DPD) estimators: Dynamic Fixed Effects (DFE)[5], Anderson-Hsaio (AH)[6], Difference GMM (DGMM)[7], and System GMM (SGMM)[8]. The results are reported in TABLE 6 and again consist of three panels: (i) the median estimated *LRP* values, (ii) the 90 percent empirical sample ranges of *LRP* values, and (iii) the Type I error rates. The performance results for each of the four DPD estimators are reported by columns.

Substantial biases in the *LRP* estimates are evident for all values of β_y, with the biases again getting worse as β_y gets closer to one. For example, when $\beta_y = 0.80$, the median *LRP* estimates for the DFE, AH, DGMM, and SGMM estimators are 3.1, 2.4, 3.4, and 4.6 (compared to a true value of 5). The 90 percent empirical sample ranges perform erratically. The 90 percent intervals for the DFE and DGMM estimators do not contain the true value. The 90 percent interval for the AH estimator does contain the true value, but it is very wide, ranging from -14.4 to 18.3.

In contrast, the SGMM estimator performs relatively well, with its median value coming closest to the true *LRP*, and the 90 percent sample range including the true value. Unfortunately, the relative performance of the SGMM estimator declines dramatically as β_y gets close to one. When $\beta_y = 0.95$, the median estimated value of *LRP* is 58.7 (compared to its true value of 20), and the associated 90 percent interval range extends from -507.2 to 597.5. This behaviour is characteristic of the SGMM estimator.

[5] See Blackburne and Frank (2007). To obtain DFE estimates of *LRP* for TABLE 6, we estimate Equation (7) using OLS with fixed effects, and calculate *LRP* according to Equation (2).
[6] See Anderson, and Hsiao (1981). To obtain AH estimates of *LRP* for TABLE 6, we estimate Equation (7) using the *xtivreg* procedure in Stata, and calculate *LRP* according to Equation (2).
[7] See Holtz-Eakin et al., (1988) and Arellano and Bond (1991). To obtain DGMM estimates of *LRP* for TABLE 6, we estimate Equation (7) using the *xtabond* procedure in Stata, and calculate *LRP* according to Equation (2).
[8] See Blundell and Bond (1998). To obtain SGMM estimates of *LRP* for TABLE 6, we estimate Equation (7) using the *xtdpdsys* procedure in Stata, and calculate *LRP* according to Equation (2).

Finally, we note that the Type I error rates are very poor across the board, with size distortions sufficiently large so as to render hypothesis testing useless.

Nonstationarity and cointegration. TABLE 7 explores the consequences of nonstationarity in both univariate time series and panel data settings. The first three columns use the univariate time series DGP of Equation (5) with *T = 10, 50,* and *1000*, respectively. The next two columns use the panel DGP of Equation (7) with (*N=50*, *T=10*) and (*N=50*, *T=25*). The top part of the table reports results when $\beta_y = 0.60$ and *LRP = 2.5*. For the bottom part of the table, $\beta_y = 0.90$ and *LRP = 10.* Moving across rows allows one to identify the impact of increasing the length of the time series. A comparison of the top and bottom panels allows one to identify the effect of increasing β_y.

In the univariate time series experiments, we let $x_t = \rho_x x_{t-1} + \nu_t$, $\nu_t \sim NID(0,1)$. In the stationary case (Case a), $\rho_x = 0$. In the nonstationary case (Case b), $\rho_x = 1$ and *x* and *y* are cointegrated. We follow a similar procedure for the panel data experiments. Comparison of Cases a) and b) allow us to study the effect of nonstationarity on estimator performance. Finally, for our estimator, we use Dynamic OLS (DOLS) for the univariate time series, and Panel Dynamic OLS (Panel DOLS) for the panel data.[9] These estimators are designed to estimate long-run relationships when data are cointegrated. TABLE 7 reports the results, using the tri-partite performance measures of (i) median value of *LRP,* (ii) 90 percent empirical sample range of estimated *LRP* values, and (iii) Type I error rate.

In all the experiments, and on all three performance measures, the DOLS estimator produces better results when $\rho_x = 1$ (Case b) compared to $\rho_x = 0$ (Case a). This is to be expected, as the DOLS estimator is designed for cointegrated data. However, the performance

[9] To obtain DOLS estimates of *LRP* for the univariate time series section of TABLE 7, we regress *y* on *x* using the user-written Stata program *cointreg* (see Wang and Wu, 2012). To obtain Panel DOLS estimates of *LRP* for the panel data section of TABLE 7, we regress *y* on *x* using the user-written Stata program *xtdolshm* (see Kao and Chiang, 2002).

of the estimator declines as β_y get close to one. For example, in Column (5), when $\beta_y = 0.60$ and (*N* = *50*, *T* = *25*), the median estimated value of *LRP* = 2.5, equal to its true value. Further the 90 percent sample range is tightly clustered around the true value, ranging from 2.4 to 2.5. However, when $\beta_y = 0.90$, the median estimated value is 8.9, and the 90 percent interval no longer contains the true value. In all the experiments, hypothesis testing continues to be unreliable, with Type I error rates far from their true values, even in those cases where the DOLS estimator performs well on other dimensions.

The experiments of TABLES 6 and 7 highlight a number of issues for estimation of *LRP*. First, the performance of different panel data estimators vary widely, with some performing consistently better than others (for example, compare the 90 percent estimation intervals for the DFE and Anderson-Hsaio estimators in TABLE 6). Second, the relative performances of the respective estimators can differ substantially depending on the value of β_y. This can be seen by comparing the relative performances of the SGMM estimator in TABLE 6 when $\beta_y = 0.60$ and $\beta_y = 0.95$. Third, persistence in the explanatory variable affects estimator performance, as evidenced by Cases a) and b) in TABLE 7. And, lastly, hypothesis testing is generally unreliable.

4. ESTIMATION OF *LRP* USING PANEL DATA

The final set of experiments is designed to compare the relative performances of a large number of panel data estimators. The ultimate goal is to develop a set of recommendations about which estimator(s) are "best" in a given research situation. While these experiments are designed to move closer to this goal, the results should be interpreted as preliminary, as they abstract away from a large number of issues. We study the following panel data estimators:

A. Anderson-Hsaio (AH)
B. Difference GMM (DGMM)
C. System GMM (SGMM)

D. Mean Group estimation of the ARDL model (MG1)[10]
E. Correlated Effect Mean Group (CCEMG)[11]
F. Augmented Mean Group (AMG)[12]
G. Dynamic Fixed Effects (DFE)[13]
H. Mean Group estimation of the growth model (MG2)[14]
I. Panel Dynamic OLS (DOLS)

The experiments use the DGP of Equation (7) where *N=50* and *T=25*. Because it is common in the macro panel literature to transform annual data into period averages, we also consider the case where *N = 50* and *T=5*, where the 25 years of annual data have been collapsed into 5 observations of 5-year averages. We conduct separate experiments for the cases $\beta_y = 0.60$ and $\beta_y = 0.90$, and for different degrees of persistence in x, $x_{it} = \rho_x x_{i,t-1} + a_i + u_{it}$, $a_i, \nu_t \sim NID(0,1)$, where $\rho_x = 0, 0.5, 0.90, 0.95, \text{and } 1$. Finally, rather than reporting 90 percent intervals of *LRP* estimates, we report the associated mean squared error (MSE) values.[15] The results of our experiments are presented in TABLES 8A through 8I.[16] A number of findings are worth highlighting.

For all estimators other than the Anderson-Hsaio estimator (TABLE8A), taking 5-year averages rather than using annual data results in less efficient estimates of *LRP*.[17] This is true

[10] See Pesaran and Smith (1995). To obtain MG1 estimates of *LRP* for TABLE 8, we estimate Equation (7) using the user-written, Stata program *xtmg* (see Eberhardt, 2012), and calculate *LRP* according to Equation (2).
[11] See Pesaran (2006). To obtain CCEMG estimates of *LRP* for TABLE 8, we estimate Equation (7) using the user-written, Stata program *xtmg* (see Eberhardt, 2012), and calculate *LRP* according to Equation (2).
[12] See Eberhardt (2012). To obtain AMG estimates of *LRP* for TABLE 8, we estimate Equation (7) using the user-written, Stata program *xtmg* (see Eberhardt, 2012), and calculate *LRP* according to Equation (2).
[13] To obtain DFE estimates of *LRP* for TABLE 8, we estimate the panel version of Equation (3.a) using the user-written, Stata program *xtpmg* (see Blackburne and Frank, 2007), and calculate *LRP* as $\hat{\theta}$.
[14] To obtain MG2 estimates of *LRP* for TABLE 8, we estimate the panel version of Equation (3.a) using the user-written, Stata program *xtpmg* (see Blackburne and Frank, 2007), and calculate *LRP* as $\hat{\theta}$.
[15] We note that some of the MSE values are exceptionally large. When the βy values are imprecisely estimated, some of the estimates may be close to 1. This makes the denominator of the LRP close to zero, which can cause MSE values to attain very large size.
[16] TABLES 8A to 8I are based on 1,000 simulations rather than 10,000 simulations because of the computing time required to run 10,000 simulations. However, our previous analyses had indicated that there was generally little to be gained by increasing the number of simulations past 1000.
[17] We note that panel DOLS estimates cannot be calculated when T=5 because the use of leads and lags does not allow the estimator to be computed.

both for different values of β_y, and for different degrees of persistence in *x*. We note that these findings are consistent with Ditzen and Gundlach (2016). For example, when $\beta_y = 0.60$ and $\rho_x = 0$, Difference GMM produces a MSE error value of 0.043 when the panel data consist of annual observations (TABLE 8B). However, when the same data are transformed into 5 observations of 5-year averaged data, the associated MSE increases to 0.172. Because the AH estimator is dominated by other estimators, the subsequent discussion focusses on the annual (*T=25*) results.

Another finding from TABLES 8A to 8I is that greater persistence in *x* is generally associated with less biased and more efficient *LRP* estimates. For example, for the Difference GMM results in TABLE 8B, when $\beta_y = 0.60$ -- so that the corresponding true value of *LRP* = 2.5 -- the median *LRP* values rises from 2.3 when $\rho_x = 0$, to 2.5 when $\rho_x = 1$. The corresponding MSE values fall from 0.043 to 0.004. A similar, monotonic improvement for increases in ρ_x is evident when $\beta_y = 0.90$. We attribute this result to the fact that Var(x_{it}) increases with ρ_x, resulting in more precise estimates of model parameters. This is true even when *x* is a nonstationary, random walk process.

Finally, of the nine estimators whose performances are reported in TABLES 8A through 8I, three stand out on the basis of mean squared error (MSE): Difference GMM (TABLE 8B), System GMM (TABLE 8C), and Dynamic Fixed Effects (TABLE 8G). For example, for panel datasets of size (*N=50*, *T=25*), when $\beta_y = 0.90$ and *LRP = 10*, and there is no serial correlation in x ($\rho_x = 0$), the MSE values for the DGMM, SGMM, and DFE estimators are 11.8, 14.5, and 10.4, respectively. The MSEs for the other estimators are 194,065.6 (AH), 21.1 (MG1), 29.2 (CCEMG), 28.5 (AMG), 36,131.7 (MG2), and 81.9 (DOLS).

Stated differently, the corresponding 90 percent sample ranges are:

- DGMM: (5.3 – 8.2)

- SGMM: (9.2 – 17.4)
- DFE: (5.8 – 8.0)

compared to:

- AH: (-14.7 – 18.0)
- MG1: (4.6 – 6.4)
- CCEMG: (3.7 – 5.8)
- AMG: (3.8 – 5.8)
- MG2: (-6.0 – 23.8)
- DOLS: (-0.1 – 2.0)[18]

When β_y is smaller and there is moderate serial correlation in *x*, all the panel data estimators perform substantially better, though DGMM, SGMM, and DFE are generally superior. Below are the corresponding MSE / (90 percent sample ranges) for the respective estimators when $\beta_y = 0.60$, *LRP = 2.5*, and $\rho_x = 0.5$:

- DGMM: 0.019 / (2.2 – 2.6)
- SGMM: 0.013 / (2.3 – 2.7)
- DFE: 0.020 / (2.3 – 2.5)

compared to:

- AH: 2500 / (1.1 – 8.0)
- MG1: 0.055 / (2.1 – 2.4)
- CCEMG: 0.112 / (2.0 – 2.4)
- AMG: 0.096 / (2.0 – 2.4)
- MG2: 0.017 / (2.3 – 2.6)
- DOLS: 1.218 / (1.1 – 1.7)

Unfortunately, as the bottom panels of TABLES 8A through 8I attest, hypothesis testing is still unreliable for all the estimators in almost all circumstances.

Given that Fieller's method provided substantial improvements for univariate time series, we investigate whether it can provide similar improvements for panel data. TABLE 9 reports our results, focusing on the DGMM, SGMM, and DFE estimators. For each of these, the top panel reproduces the previously reported Type I error rates, while the bottom panel

[18] While it might be unfair to include DOLS in this comparison of estimators with stationary data, we note that DOLS does not do appreciably better when *x* and *y* are nonstationary and cointegrated ($\rho_x = 1$).

reports Type I error rates using Fieller's method. The results are disappointing. There is, at best, only minor improvement from adopting Fieller's method – far less than the amount of improvement that would be necessary to make hypothesis testing serviceable.

In summary, our analysis identifies three panel data estimators that perform substantially better than the others. Difference GMM, System GMM, and Dynamic Fixed Effects were generally more efficient than other panel data estimators across a wide variety of data environments. Our results suggest that when there is moderate persistence in both the dependent and explanatory variables, these estimators may estimate *LRP* sufficiently precisely so as to be useful in practical research situations. That being said, our analysis has identified a number of problematic data scenarios. When values of β_y are close to one, none of the estimators perform very well. Further, we find that hypothesis testing is generally unreliable.

A caveat to our results is that the experiments ignore a number of factors that would be expected to influence the relative performance of the panel data estimators. For example, the various mean group estimators should perform relatively better when slope coefficients differ across cross-sectional units. However, our experiments imposed coefficient homogeneity. Likewise, the Common Correlated Effects Mean Group (CCEMG) and Augmented Mean Group (AMG) estimators should perform relatively better when common factors induce cross-sectional dependence across observations. Our experiments abstracted away from this issue. As a result, our findings should be viewed as preliminary. Further testing in more diverse data environments is called for.

5. CONCLUSION

A common practice in economic research is to calculate long-run impacts based on the estimated coefficients from a model involving one or more lagged dependent variables. For example, given the ARDL(1,1) model, $y_t = \beta_0 + \beta_x x_t + \beta_y y_{t-1} + u_t$, the long-run effect of *x* on *y*, also known as the long-run propensity (*LRP*) of *x*, is estimated by $LRP = \hat{\beta}_x / (1 - \hat{\beta}_y)$.

When this model is recast as a growth regression, such as $\Delta y_t = \beta_0 + \delta(y_{t-1} - \theta x_t) + u_t$, or $\Delta y_t = \beta_0 + \delta(y_{t-1} - \theta x_{t-1}) + \beta_x \Delta x_t + u_t$, the corresponding estimate of *LRP* is given by $\hat{\theta}$. This note uses Monte Carlo experiments to demonstrate that this practice can be hazardous. It often fails to produce reliable estimates. Biases can be substantial, sample ranges very wide, and hypothesis tests can be rendered useless in realistic data environments.

The reason for this poor performance is threefold. First, estimates of the coefficient of a lagged dependent variable are downwardly biased in finite samples (Hurwicz, 1950; Phillips, 1977). Second, small biases in the estimate of β_y will be substantially magnified in estimates of *LRP* when the value of β_y is close to one. Rewriting the specification as a growth equation does not allow one to avoid this problem. Finally, the statistical distribution associated with estimates of *LRP* is heavy tailed and complicated. While some of these problems have previously been identified in the literature, the widespread use of these procedures suggests that researchers may not be aware of the practical significance of these problems. By demonstrating their extent and severity, it is hoped that this study will stimulate further research towards the estimation of long-run impacts.

Our study also makes some contributions about what to do and what not to do when it comes to estimating long-run impacts. First, our experiments indicate that the practice of data averaging – where a researcher takes, say, 25 years of annual data and transforms these into five observations of 5-year averaged data – generally results in less efficient estimates of *LRP*¸ and that the loss in efficiency can be substantial. Second, we demonstrate that some estimators perform systematically better than others. While our results are not sufficiently extensive to be prescriptive, they do indicate that further research comparing estimators in a wider variety of data environments could result in recommendations for applied researchers. Third, our results indicate that neither (i) Fieller's method, nor (ii) jackknifing, nor (iii) indirect inference provide general solutions for estimating *LRP* with panel data. By eliminating these possibilities, it is hoped that researchers can concentrate on other, more promising directions. Lastly, we are able to show that conventional hypothesis testing is generally unreliable when testing hypotheses about *LRP*. Other approaches, such as bootstrapping, should be explored.

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TABLE 1
Demonstration of Finite-Sample Bias in OLS Estimation of an AR Model

True Value of $\boldsymbol{\beta_y}$	***SAMPLE SIZE***				
	T=10	***T=30***	***T=50***	***T=100***	***T=1000***
	Average Estimated Value of $\boldsymbol{\beta_y}$				
0.60	0.34	0.51	0.54	0.57	0.60
0.70	0.41	0.60	0.64	0.67	0.70
0.80	0.48	0.68	0.73	0.77	0.80
0.90	0.53	0.77	0.82	0.86	0.90
0.95	0.55	0.81	0.86	0.91	0.95
	Type I Error Rate (H_0: $\boldsymbol{\beta_y}$ = true value)				
0.60	0.07	0.06	0.06	0.05	0.05
0.70	0.07	0.07	0.06	0.06	0.05
0.80	0.09	0.09	0.08	0.07	0.05
0.90	0.13	0.12	0.10	0.08	0.05
0.95	0.16	0.16	0.15	0.11	0.06

SOURCE: 10,000 replications were run for each experiment. The experiments are described in more detail in Section 2 of the text.

TABLE 2
Estimation of *LRP* in an ARDL Model: OLS

True Value of β_y / True Value of LRP	***SAMPLE SIZE*** ***T=10***	***T=50***	***T=1000***
	Median Value of Estimated LRP		
0.60 / 2.5	1.9	2.4	2.5
0.70 / 3.3	2.1	3.1	3.3
0.80 / 5	2.5	4.4	5.0
0.90 / 10	2.8	7.5	9.9
0.95 / 20	2.8	11.2	19.4
	90 Percent Empirical Sample Range for Estimated LRP		
0.60 / 2.5	0.4 — 6.5	1.5 — 3.6	2.3 — 2.7
0.70 / 3.3	0.3 — 9.0	1.9 — 5.0	3.0 — 3.7
0.80 / 5	-0.6 — 13.4	2.4 — 8.1	4.4 — 5.6
0.90 / 10	-11.3 — 20.4	3.3 — 19.1	8.3 — 11.6
0.95 / 20	-21.3 — 25.6	3.8 — 44.4	15.3 — 24.8
	Type I Error Rate (H_0: LRP = true value)		
0.60 / 2.5	0.23	0.09	0.05
0.70 / 3.3	0.29	0.12	0.05
0.80 / 5	0.38	0.17	0.06
0.90 / 10	0.54	0.27	0.06
0.95 / 20	0.67	0.40	0.09

SOURCE: 10,000 replications were run for each experiment. The experiments are described in more detail in Section 2 of the text.

TABLE 3
Testing *LRP* in an ARDL Model: Wald Versus Fieller's Test

True Value of β_y / True Value of LRP	***SAMPLE SIZE***		
	T=10	***T=50***	***T=1000***
Type I Error Rate (H_0: LRP = true value) --- Wald test			
0.60 / 2.5	0.23	0.09	0.05
0.70 / 3.3	0.29	0.12	0.05
0.80 / 5	0.38	0.17	0.06
0.90 / 10	0.54	0.27	0.06
0.95 / 20	0.67	0.40	0.09
Type I Error Rate (H_0: LRP = true value) – Fieller test			
0.60 / 2.5	0.10	0.06	0.05
0.70 / 3.3	0.12	0.06	0.05
0.80 / 5	0.13	0.07	0.05
0.90 / 10	0.15	0.09	0.05
0.95 / 20	0.17	0.11	0.06

SOURCE: 10,000 replications were run for each experiment. The experiments are described in more detail in Section 2 of the text.

TABLE 4
Comparison of Jackknifing (JK) and Indirect Inference (II)
with OLS Estimates of β_y in an ARDL Model

True Value of β_y	*SAMPLE SIZE* *T=10*	*T=30*	*T=50*	*T=100*	*T=1000*
	OLS estimate of β_y				
0.60	0.46	0.55	0.57	0.59	0.60
0.70	0.54	0.64	0.67	0.68	0.70
0.80	0.62	0.74	0.76	0.78	0.80
0.90	0.70	0.83	0.86	0.88	0.90
0.95	0.73	0.87	0.90	0.93	0.95
	Jackknifing (JK) estimate of β_y				
0.60	0.57	0.60	0.60	0.60	0.60
0.70	0.66	0.70	0.70	0.70	0.70
0.80	0.76	0.80	0.80	0.80	0.80
0.90	0.85	0.90	0.90	0.90	0.90
0.95	0.89	0.95	0.95	0.95	0.95
	Indirect Inference (II) estimate of β_y				
0.60	0.60	0.60	0.60	0.60	0.60
0.70	0.69	0.70	0.70	0.70	0.70
0.80	0.78	0.80	0.80	0.80	0.80
0.90	0.86	0.90	0.90	0.90	0.90
0.95	0.89	0.94	0.95	0.95	0.95

SOURCE: 50,000 replications were run for T=10 cases and 10,000 replications were run for all other experiments. The experiments are described in more detail in Section 2 of the text.

TABLE 5
Comparison of Jackknifing (JK) and Indirect Inference (II) with OLS Estimates of *LRP* in an ARDL Model

True Value of β_y / True Value of LRP	*SAMPLE SIZE* T=10	T=50	T=1000
90 Percent Empirical Sample Range for Estimated LRP -- OLS			
0.60 / 2.5	0.4 — 6.5	1.5 — 3.6	2.3 — 2.7
0.70 / 3.3	0.3 — 9.0	1.9 — 5.0	3.0 — 3.7
0.80 / 5	-0.6 — 13.4	2.4 — 8.1	4.4 — 5.6
0.90 / 10	-11.3 — 20.4	3.3 — 19.1	8.3 — 11.6
0.95 / 20	-21.3 — 25.6	3.8 — 44.4	15.3 — 24.8
90 Percent Empirical Sample Range for Estimated LRP -- JK			
0.60 / 2.5	-9.5 – 12.9	1.5 – 4.4	2.3 – 2.7
0.70 / 3.3	-13.3 – 15.3	1.9 – 7.2	3.0 – 3.7
0.80 / 5	-17.3 – 17.5	2.3 – 15.9	4.4 – 5.7
0.90 / 10	-18.9 – 19.4	-32.4 – 46.3	8.4 – 12.1
0.95 / 20	-19.7 – 19.6	-71.5 – 87.0	15.7 – 26.7
90 Percent Empirical Sample Range for Estimated LRP -- II			
0.60 / 2.5	-11.2 – 14.9	1.6 – 4.0	2.3 – 2.7
0.70 / 3.3	-19.1 – 20.8	2.0 – 6.0	3.0 – 3.7
0.80 / 5	-26.5 – 28.1	2.7 – 12.4	4.4 – 5.7
0.90 / 10	-34.8 – 33.0	-30.7 – 54.5	8.5 – 12.0
0.95 / 20	-37.7 – 34.7	-132.1 –143.1	15.8 – 26.1

SOURCE: 50,000 replications were run for T=10 cases and 10,000 replications were run for all other experiments. The experiments are described in more detail in Section 2 of the text.

TABLE 6
Estimation of *LRP* in a DPD Model (*N=50*, *T=10*): DFE, AH, DGMM, and SGMM

True Value of β_y / True Value of LRP	*ESTIMATOR* *DFE*	*Anderson-Hsaio*	*Difference GMM*	*System GMM*
	Median Value of Estimated LRP			
0.60 / 2.5	2.0	2.4	2.2	2.4
0.70 / 3.3	2.4	2.8	2.7	3.1
0.80 / 5	3.1	2.4	3.4	4.6
0.90 / 10	4.2	1.1	4.5	11.9
0.95 / 20	6.0	0.6	7.0	58.7
	Average 90 Percent Empirical Sample Range for Estimated LRP			
0.60 / 2.5	1.7 — 2.3	1.4 — 6.3	1.8 — 2.5	2.0 — 2.8
0.70 / 3.3	2.1 — 2.9	-4.2 — 13.5	2.2 — 3.2	2.5 — 3.9
0.80 / 5	2.5 — 3.8	-14.4 — 18.3	2.6 — 4.4	3.4 — 6.5
0.90 / 10	3.3 — 5.5	-10.4 — 11.8	3.2 — 6.4	6.9 — 24.1
0.95 / 20	4.5 — 8.2	-7.3 — 9.0	4.6 — 11.3	-507.2 — 597.5
	Average Type I Error Rate (H_0: LRP = true value)			
0.60 / 2.5	0.78	0.09	0.41	0.16
0.70 / 3.3	0.92	0.15	0.59	0.20
0.80 / 5	0.99	0.27	0.81	0.21
0.90 / 10	1.00	0.60	0.97	0.07
0.95 / 20	1.00	0.76	0.96	0.02

SOURCE: 10,000 replications were run for each experiment. The experiments are described in more detail in Section 3 of the text. DFE, AH, DGMM, and SGMM stand for Dynamic Fixed Effects, Anderson-Hsaio, Difference GMM, and System GMM.

TABLE 7
Estimation of *DOLS and Panel DOLS*

Autocorrelation in x (ρ_X)	*DOLS*			*Panel DOLS*	
	T = 10 (1)	*T = 50* (2)	*T = 1000* (3)	*N = 50, T = 10* (4)	*N = 50, T = 25* (5)
	β_y = 0.60, *LRP* = 2.5				
	Median Value of Estimated LRP				
a) 0	1.7	1.9	2.0	1.0	1.0
b) 1	2.0	2.4	2.5	2.5	2.5
	90 Percent Empirical Sample Range for Estimated LRP				
a) 0	-2.2 — 5.5	0.9 — 2.9	1.8 — 2.2	0.5 — 1.5	0.7 — 1.3
b) 1	-0.7 — 4.5	2.0 — 2.7	2.5 — 2.5	2.4 — 2.5	2.4 — 2.5
	Type I Error Rate (H_0: LRP = true value)				
a) 0	0.31	0.41	1.00	1.00	1.00
b) 1	0.35	0.42	0.44	0.00	0.18
	β_y = 0.90, *LRP* = 10				
	Median Value of Estimated LRP				
a) 0	1.3	2.1	2.7	1.0	1.0
b) 1	2.7	5.7	9.7	9.0	8.9
	90 Percent Empirical Sample Range for Estimated LRP				
a) 0	-3.6 — 6.2	0.3 — 4.3	2.2 — 3.2	-0.9 — 2.9	0.0 — 2.1
b) 1	-5.4 — 11.5	2.3 — 9.8	9.0 — 10.0	8.5 — 9.5	8.5 — 9.3
	Type I Error Rate (H_0: LRP = true value)				
a) 0	0.88	1.00	1.00	1.00	1.00
b) 1	0.80	0.89	0.91	0.82	0.99

SOURCE: 10,000 replications were run for each experiment. The experiments are described in more detail in Section 3 of the text. AH, DGMM, and SGMM stand for Anderson-Hsaio, Difference GMM, and System GMM.

TABLE 8A
Comparison of Alternative Panel Data Estimators: Anderson-Hsaio (AH)

	β_y = 0.60, *LRP* = 2.5		β_y = 0.90, *LRP* = 10	
Autocorrelation in x (ρ_X)	***Annual (T=25)***	***5-Year Averages (T=5)***	***Annual (T=25)***	***5-Year Averages (T=5)***
		Median Value of Estimated LRP		
0	2.5	2.6	1.8	0.2
0.50	2.3	3.0	4.4	0.7
0.90	2.5	2.5	9.9	4.4
0.95	2.4	2.2	10.0	5.6
1	2.5	2.2	10.0	7.8
		Mean Squared Error of LRP		
0	1.033	0.993	194,064.560	9,368.067
0.50	2,500.213	1.130	154,491.790	95.865
0.90	84.106	0.172	8.099	31.655
0.95	0.477	0.220	3.007	19.470
1	0.213	0.123	1.019	5.309
		Type I Error Rate (H_0: LRP = true value)		
0	0.06	0.04	0.48	0.96
0.50	0.13	0.00	0.26	0.94
0.90	0.08	0.08	0.02	0.98
0.95	0.07	0.26	0.01	0.98
1	0.04	0.38	0.00	0.81

SOURCE: 1000 replications were run for each experiment. The experiments are described in more detail in Section 4 of the text.

TABLE 8B
Comparison of Alternative Panel Data Estimators: Difference GMM (DGMM)

	β_y = 0.60, *LRP* = 2.5		β_y = 0.90, *LRP* = 10	
Autocorrelation in x (ρ_X)	***Annual (T=25)***	***5-Year Averages (T=5)***	***Annual (T=25)***	***5-Year Averages (T=5)***
	Median Value of Estimated LRP			
0	2.3	2.3	6.6	4.1
0.50	2.4	2.7	8.0	5.5
0.90	2.5	2.7	9.6	9.1
0.95	2.5	2.6	9.7	7.2
1	2.5	2.4	9.9	8.3
	Mean Squared Error of LRP			
0	0.043	0.172	11.806	34.971
0.50	0.019	0.132	4.657	32.235
0.90	0.005	0.078	0.383	7.308
0.95	0.005	0.041	0.244	8.892
1	0.004	0.025	0.085	3.222
	Type I Error Rate (H_0: LRP = true value)			
0	0.26	0.10	0.92	0.91
0.50	0.19	0.04	0.66	0.60
0.90	0.11	0.38	0.16	0.30
0.95	0.11	0.13	0.12	0.79
1	0.10	0.20	0.07	0.94

SOURCE: 1000 replications were run for each experiment. The experiments are described in more detail in Section 4 of the text.

TABLE 8C
Comparison of Alternative Panel Data Estimators: System GMM (SGMM)

	β_y = 0.60, *LRP* = 2.5		β_y = 0.90, *LRP* = 10	
Autocorrelation in x (ρ_X)	***Annual (T=25)***	***5-Year Averages (T=5)***	***Annual (T=25)***	***5-Year Averages (T=5)***
		Median Value of Estimated LRP		
0	2.5	2.7	12.4	-82.1
0.50	2.5	3.0	11.4	-75.9
0.90	2.5	2.9	10.4	20.0
0.95	2.5	2.8	10.3	13.9
1	2.5	2.6	10.1	10.9
		Mean Squared Error of LRP		
0	0.024	0.407	14.542	8,052,204.400
0.50	0.013	0.379	5.729	12,309,295.000
0.90	0.005	0.172	0.612	124.364
0.95	0.004	0.098	0.304	18.238
1	0.003	0.015	0.099	1.158
		Type I Error Rate (H_0: LRP = true value)		
0	0.10	0.13	0.22	0.09
0.50	0.11	0.41	0.22	0.08
0.90	0.11	0.93	0.22	1.00
0.95	0.10	0.85	0.23	1.00
1	0.18	0.56	0.30	0.85

SOURCE: 1000 replications were run for each experiment. The experiments are described in more detail in Section 4 of the text.

TABLE 8D
Comparison of Alternative Panel Data Estimators: Mean Group – ARDL (MG1)

	β_y = 0.60, *LRP* = 2.5		β_y = 0.90, *LRP* = 10	
Autocorrelation in x (ρ_X)	***Annual (T=25)***	***5-Year Averages (T=5)***	***Annual (T=25)***	***5-Year Averages (T=5)***
	Median Value of Estimated LRP			
0	2.2	3.8	5.4	13.3
0.50	2.3	3.0	6.8	11.2
0.90	2.4	2.6	8.7	10.3
0.95	2.4	2.6	8.9	10.2
1	2.4	2.6	9.2	10.1
	Mean Squared Error of LRP			
0	0.113	2.976	21.087	6,235.493
0.50	0.055	0.370	10.566	43.809
0.90	0.015	0.030	2.046	1.258
0.95	0.012	0.036	1.434	0.850
1	0.009	0.018	0.852	0.539
	Type I Error Rate (H_0: LRP = true value)			
0	0.74	0.40	1.00	0.01
0.50	0.60	0.34	0.99	0.02
0.90	0.33	0.16	0.71	0.05
0.95	0.29	0.15	0.63	0.04
1	0.26	0.13	0.48	0.03

SOURCE: 1000 replications were run for each experiment. The experiments are described in more detail in Section 4 of the text.

TABLE 8E
Comparison of Alternative Panel Data Estimators: Common Correlated Effect Mean Group (CCEMG)

	β_y = 0.60, *LRP* = 2.5		β_y = 0.90, *LRP* = 10	
Autocorrelation in x (ρ_X)	***Annual (T=25)***	***5-Year Averages (T=5)***	***Annual (T=25)***	***5-Year Averages (T=5)***
		Median Value of Estimated LRP		
0	2.1	3.5	4.6	8.5
0.50	2.2	2.9	5.6	9.4
0.90	2.3	2.7	7.0	9.7
0.95	2.3	2.6	7.2	9.9
1	2.3	2.6	7.6	9.6
		Mean Squared Error of LRP		
0	0.173	5,349.265	29.167	122,592.010
0.50	0.112	494.082	19.164	127,580.940
0.90	0.049	18.470	9.310	2,682.697
0.95	0.045	474,147.790	8.023	12,517.638
1	0.038	8.146	6.631	6,057.932
		Type I Error Rate (H_0: LRP = true value)		
0	0.82	0.03	1.00	0.12
0.50	0.80	0.04	1.00	0.10
0.90	0.67	0.04	0.97	0.12
0.95	0.66	0.03	0.93	0.10
1	0.60	0.04	0.92	0.11

SOURCE: 1000 replications were run for each experiment. The experiments are described in more detail in Section 4 of the text.

TABLE 8F
Comparison of Alternative Panel Data Estimators: Augmented Mean Group (AMG)

	β_y = 0.60, *LRP* = 2.5		β_y = 0.90, *LRP* = 10	
Autocorrelation in x (ρ_X)	***Annual (T=25)***	***5-Year Averages (T=5)***	***Annual (T=25)***	***5-Year Averages (T=5)***
	Median Value of Estimated LRP			
0	2.1	3.8	4.6	12.2
0.50	2.2	2.9	5.9	11.3
0.90	2.3	2.6	7.9	10.3
0.95	2.4	2.6	8.2	10.2
1	2.4	2.6	8.6	10.1
	Mean Squared Error of LRP			
0	0.183	128.699	28.497	3,512.523
0.50	0.096	3.787	16.751	3,871.171
0.90	0.033	0.314	5.097	11.598
0.95	0.028	0.161	3.724	10.221
1	0.022	0.207	2.536	9.747
	Type I Error Rate (H_0: LRP = true value)			
0	0.91	0.07	1.00	0.04
0.50	0.82	0.10	1.00	0.04
0.90	0.60	0.07	0.91	0.05
0.95	0.56	0.07	0.86	0.04
1	0.50	0.07	0.75	0.05

SOURCE: 1000 replications were run for each experiment. The experiments are described in more detail in Section 4 of the text.

TABLE 8G
Comparison of Alternative Panel Data Estimators: Dynamic Fixed Effects (DFE)

	β_y = 0.60, *LRP* = 2.5		β_y = 0.90, *LRP* = 10	
Autocorrelation in x (ρ_X)	***Annual (T=25)***	***5-Year Averages (T=5)***	***Annual (T=25)***	***5-Year Averages (T=5)***
	Median Value of Estimated LRP			
0	2.3	1.9	6.8	4.7
0.50	2.4	2.4	8.2	7.7
0.90	2.5	2.7	9.6	14.2
0.95	2.5	2.7	9.7	13.2
1	2.5	2.6	9.9	11.5
	Mean Squared Error of LRP			
0	0.053	0.374	10.449	28.082
0.50	0.020	0.042	3.675	6.586
0.90	0.003	0.052	0.284	20.482
0.95	0.003	0.038	0.176	12.384
1	0.002	0.016	0.060	2.511
	Type I Error Rate (H_0: LRP = true value)			
0	0.42	0.67	0.98	0.98
0.50	0.26	0.09	0.79	0.45
0.90	0.10	0.61	0.23	0.91
0.95	0.10	0.64	0.18	0.96
1	0.09	0.47	0.10	0.81

SOURCE: 1000 replications were run for each experiment. The experiments are described in more detail in Section 4 of the text.

TABLE 8H
Comparison of Alternative Panel Data Estimators: Mean Group – Growth (MG2)

	β_y = 0.60, *LRP* = 2.5		β_y = 0.90, *LRP* = 10	
Autocorrelation in x (ρ_X)	***Annual (T=25)***	***5-Year Averages (T=5)***	***Annual (T=25)***	***5-Year Averages (T=5)***
		Median Value of Estimated LRP		
0	2.41	2.34	7.74	3.45
0.50	2.42	2.86	8.93	4.58
0.90	2.46	2.98	10.00	10.83
0.95	2.46	2.93	10.00	11.37
1	2.47	2.84	10.01	11.23
		Mean Squared Error of LRP		
0	0.034	842.337	36,131.667	8,328.382
0.50	0.017	3,981.552	8,703.481	13,259.008
0.90	0.007	2,063.033	6,481.592	52,157.935
0.95	0.006	101.173	15,903.411	1,728,054.700
1	0.004	25.793	28.286	25,414.063
		Type I Error Rate (H_0: LRP = true value)		
0	0.14	0.04	0.26	0.33
0.50	0.14	0.05	0.12	0.15
0.90	0.11	0.35	0.06	0.01
0.95	0.11	0.35	0.05	0.03
1	0.10	0.33	0.07	0.03

SOURCE: 1000 replications were run for each experiment. The experiments are described in more detail in Section 4 of the text.

TABLE 8I
Comparison of Alternative Panel Data Estimators: Dynamic OLS (DOLS)

	β_y = 0.60, *LRP* = 2.5		β_y = 0.90, *LRP* = 10	
Autocorrelation in x (ρ_X)	***Annual (T=25)***	***5-Year Averages (T=5)***	***Annual (T=25)***	***5-Year Averages (T=5)***
	Median Value of Estimated LRP			
0	1.0	---	1.0	---
0.50	1.4	---	2.0	---
0.90	2.3	---	5.7	---
0.95	2.4	---	6.7	---
1	2.4	---	7.4	---
	Mean Squared Error of LRP			
0	2.301	---	81.870	---
0.50	1.218	---	64.348	---
0.90	0.080	---	19.012	---
0.95	0.031	---	11.182	---
1	0.013	---	7.171	---
	Type I Error Rate (H_0: LRP = true value)			
0	1.00	---	1.00	---
0.50	1.00	---	1.00	---
0.90	0.89	---	1.00	---
0.95	0.74	---	1.00	---
1	0.57	---	1.00	---

SOURCE: 1000 replications were run for each experiment. The experiments are described in more detail in Section 4 of the text.

TABLE 9
Does Fieller's Method Improve Hypothesis Testing in Panel Data Context?

Autocorrelation in x (ρ_X)	*True Value of β_y*				
	0.60	***0.70***	***0.80***	***0.90***	***0.95***
DYNAMIC FIXED EFFECTS					
Type I Error Rate (H_0: LRP = true value) --- Wald test					
0.60	0.42	0.60	0.81	0.98	0.98
0.70	0.26	0.36	0.53	0.80	0.88
0.80	0.10	0.13	0.18	0.24	0.29
0.90	0.10	0.12	0.14	0.17	0.20
0.95	0.09	0.09	0.11	0.10	0.12
Type I Error Rate (H_0: LRP = true value) --- Fieller test					
0.60	0.37	0.55	0.76	0.95	0.96
0.70	0.23	0.32	0.48	0.73	0.81
0.80	0.09	0.11	0.16	0.21	0.23
0.90	0.10	0.11	0.13	0.16	0.17
0.95	0.08	0.08	0.10	0.09	0.10
DIFFERENCE GMM					
Type I Error Rate (H_0: LRP = true value) --- Wald test					
0.60	0.26	0.40	0.62	0.91	0.87
0.70	0.19	0.26	0.39	0.66	0.75
0.80	0.11	0.11	0.14	0.14	0.20
0.90	0.11	0.12	0.13	0.12	0.14
0.95	0.10	0.09	0.09	0.07	0.07
Type I Error Rate (H_0: LRP = true value) --- Fieller test					
0.60	0.22	0.34	0.53	0.84	0.75
0.70	0.16	0.21	0.34	0.56	0.58
0.80	0.10	0.10	0.11	0.11	0.15
0.90	0.09	0.10	0.10	0.10	0.10
0.95	0.10	0.08	0.08	0.06	0.07

Autocorrelation in x (ρ_X)	*True Value of β_y*				
	0.60	***0.70***	***0.80***	***0.90***	***0.95***
			SYSTEM GMM		
			Type I Error Rate (H_0: LRP = true value) --- Wald test		
0.60	0.10	0.13	0.11	0.20	0.16
0.70	0.11	0.10	0.10	0.24	0.68
0.80	0.11	0.09	0.09	0.20	0.63
0.90	0.10	0.10	0.13	0.23	0.56
0.95	0.18	0.18	0.25	0.32	0.29
			Type I Error Rate (H_0: LRP = true value) --- Fieller test		
0.60	0.11	0.11	0.13	0.41	0.97
0.70	0.11	0.10	0.11	0.37	0.95
0.80	0.10	0.09	0.09	0.25	0.69
0.90	0.10	0.10	0.13	0.24	0.59
0.95	0.18	0.18	0.25	0.32	0.30

SOURCE: 10,000 replications were run for each experiment. The experiments are described in more detail in Section 2 of the text.