**DEPARTMENT OF ECONOMICS
COLLEGE OF BUSINESS AND ECONOMICS
UNIVERSITY OF CANTERBURY
CHRISTCHURCH, NEW ZEALAND**

**A Comparative Analysis of the Stabilizing Properties of
Nominal Income Growth Targeting**

by Alfred V. Guender*

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**Department of Economics, College of Business and Economics,
University of Canterbury, Private Bag 4800, Christchurch,
New Zealand**

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Abstract

Given highly persistent cost-push shocks, the relative performance of nominal income growth targeting depends critically on the size of two key parameters. Barring extreme preferences, nominal income growth targeting performs fairly well relative to commitment and pure discretion for small values of the Phillips Curve parameter.

* Department of Economics, University of Canterbury, Private Bag 4800, Christchurch, New Zealand, Tel: 64-3-364-2519; Fax: 64-3-364-2635

Email address: Alfred.Guender@Canterbury.ac.nz

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1. Introduction

This paper evaluates the performance of nominal income growth targeting in a forward-looking model and compares its stabilizing properties to those of optimal policy from the timeless perspective, standard commitment, and pure discretion. The comparative analysis reveals that given highly persistent cost-push shocks the relative performance of nominal income growth targeting depends critically on the size of the structural parameter in the Phillips Curve and society's preferences towards inflation variability.

2. Defining the Nominal Income Growth Target

Letting x_t denote the *level* of real output, we define $p_t + x_t$ as current nominal income. The output gap is given by $y_t = x_t - \bar{x}_t$ where $\bar{x}_t$ denotes the potential level of output. If k represents the growth rate of potential output, then it follows that $k = \bar{x}_t - \bar{x}_{t-1}$. After setting k equal to zero and assuming the target for nominal income growth to be equal to zero, we can state the policy rule as:

$$y_t - y_{t-1} + \pi_t = 0 \quad (1)$$

Equation (1) is similar to the policy rule under policy from the timeless perspective or a speed limit policy in the sense that the lagged output gap figures in the determination of policy. Thus nominal income growth targeting introduces inertia into the conduct of monetary policy in way similar to policy under the timeless perspective or the speed limit policy.¹

¹ See Jensen (2002) and Walsh (2003) for contributions that analyze by way of numerical solution procedures a number of targeting strategies. McCallum and Nelson (2004) evaluate nominal income growth targeting analytically but assume only white noise disturbances.

3. Evaluation of Nominal Income Growth Targeting

The forward-looking model serves as the frame of reference for the analysis conducted in this section. It consists of a Phillips Curve and an IS relation:

$$\pi_t = \beta E_t \pi_{t+1} + a y_t + u_t \quad (2)$$

$$y_t = E_t y_{t+1} - a_I r_t + v_t \quad (3)$$

y_t = the output gap

r_t = the real rate of interest

π_t = the rate of inflation.

E_t = expectation based on information available at time t

a and a_I are positive parameters.

u_t and v_t are random disturbances that follow an autoregressive process.

$$u_t = \rho u_{t-1} + \hat{u}_t \quad 0 \leq \rho < 1 \quad \hat{u}_t \sim N(0, \sigma_u^2) \quad (4)$$

$$v_t = \nu v_{t-1} + \hat{v}_t \quad 0 \leq \nu < 1 \quad \hat{v}_t \sim N(0, \sigma_v^2) \quad (5)$$

To simplify the analysis, we assume that the policymaker has complete control over the real rate of interest which serves as the policy instrument.

To determine the behavior of the rate of inflation under nominal income growth targeting, insert the policy rule into the Phillips Curve:

$$\pi_t = \frac{a}{1+a} y_{t-1} + \frac{1}{1+a} (\beta E_t \pi_{t+1} + u_t) \quad (6)$$

Combining equation (6) with equation (1) yields the reduced form equation for the output gap:

$$y_t = \frac{1}{1+a} y_{t-1} - \frac{1}{1+a} (\beta E_t \pi_{t+1} + u_t) \quad (7)$$

To calculate the expectation of the rate of inflation in period $t+1$ that appears in equations (6) and (7), we apply the minimum state variable approach:

$$y_t = \phi_{11}u_t + \phi_{12}y_{t-1} \quad (8)$$

$$\pi_t = \phi_{21}u_t + \phi_{22}y_{t-1} \quad (9)$$

Updating equation (9) and taking expectations yields:

$$E_t\pi_{t+1} = \phi_{22}y_t + \phi_{21}\rho u_t \quad (10)$$

Combining equations (8), (9), and (10) with equations (6) and (7) and applying the method of undetermined coefficients produces the following expressions:

$$\begin{aligned} \phi_{11} &= -\frac{(\beta\phi_{21}\rho + 1)}{1 + a + \beta\phi_{22}} & \phi_{12} &= \frac{1}{1 + a + \beta\phi_{22}} \\ \phi_{21} &= \frac{\beta\phi_{22}\phi_{11} + 1}{1 + a - \beta\rho} & \phi_{22} &= \frac{a}{1 + a - \beta\phi_{12}} \end{aligned} \quad (11)$$

These four expressions can in turn be solved for the four unknown coefficients that appear in the putative solutions for the two endogenous variables:

$$\begin{aligned} \phi_{11} &= \frac{-(1 + a + \beta(1 - 2\rho)) + \Lambda}{2\beta(1 - \rho(1 + a + \beta(1 - \rho)))} & \phi_{12} &= \frac{2}{1 + \beta + a + \Lambda} \\ \phi_{21} &= \frac{1 + a + \beta(1 - 2\rho) - \Lambda}{2\beta(1 - \rho(1 + a + \beta(1 - \rho)))} & \phi_{22} &= \frac{-(1 + a) + \beta + \Lambda}{2\beta} \\ \Lambda &= \sqrt{(1 + a - \beta)^2 + 4a\beta} \end{aligned} \quad (12)$$

To assess the stabilizing properties of nominal income growth targeting relative to other strategies of monetary policy, we need to define society's objectives. A representative way of expressing society's loss function is the following:

$$E[L_t] = V(y_t) + \mu V(\pi_t) \quad (13)$$

μ indicates the extent to which society cares about the variability of the rate of inflation relative to the variability of the real output gap.

Next we compute the variances of the rate of inflation and the output gap under nominal income growth targeting. They are given by equations (14) and (15):

$$V(\pi_t) = 2\phi_{11}^2 \left[\frac{(1 - \phi_{12}^2 \rho)}{(1 + \phi_{12})(1 - \phi_{12}\rho)} - \rho \left(\frac{1 - \phi_{12}}{1 - \phi_{12}\rho} \right) \right] \sigma_u^2 \quad (14)$$

$$V(y_t) = \phi_{11}^2 \left[\frac{(1 + \phi_{12}\rho)}{(1 - \phi_{12}^2)(1 - \phi_{12}\rho)} \right] \sigma_u^2 \quad (15)$$

Substituting both variances into society's loss function and choosing representative values for a , β , ρ , and σ_u^2 allows us to evaluate the performance of nominal income growth targeting. We let $\beta = 0.99$, $\rho = 0.9$, and $\sigma_u^2 = 0.000009$ but choose two different values for a . Figure 1 traces out the loss functions for $a = 0.01$ while Figure 2 depicts the loss function for $a = 0.05$ over the range $0 < \mu \leq 350$. For the purpose of comparing the performance of nominal income growth targeting (*NIGT*) to other monetary policy strategies, we have added the expected loss function under policy from the timeless perspective (*TP*), standard commitment (*SC*), and pure discretion (*DISC*). The variances of the rate of inflation and the output gap under these policies are listed in Table 1.²

² Standard Commitment refers to the scenario where the policymaker takes account of the endogeneity of expectations when minimizing the expected loss function. In sharp contrast to policy from the timeless perspective, policy under standard commitment is not inertial or history-dependent. It is best thought of as a local optimum rather than a global optimum (Clarida, Gali, and Gertler (1999)).

Table 1:

	<i>Timeless Perspective</i>	<i>Discretion</i>	<i>Standard Commitment</i>
$V(y_t) =$	$\frac{\phi_{11}^2(1+\phi_{12}\rho)}{(1-\phi_{12}\rho)(1-\phi_{12}^2)}\sigma_u^2$	$\frac{(\mu a)^2}{(1-\beta\rho+a^2\mu)^2}\sigma_u^2$	$\frac{(\mu a)^2}{((1-\beta\rho)^2+a^2\mu)^2}\sigma_u^2$
$V(\pi_t) =$	$\frac{2\theta^2\phi_{11}^2}{(1-\phi_{12}\rho)}\left[\frac{1-\phi_{12}^2\rho}{(1+\phi_{12})}-\rho(1-\phi_{12})\right]\sigma_u^2$	$\frac{1}{(1-\beta\rho+a^2\mu)^2}\sigma_u^2$	$\frac{1}{(1-\beta\rho+\frac{a^2\mu}{1-\beta\rho})^2}\sigma_u^2$

$$\text{Note: } \sigma_u^2 = \frac{1}{1-\rho^2}\sigma_{\tilde{u}}^2 \quad \phi_{11} = \frac{-\phi_{12}+\rho}{(-a\rho+\theta(1-\rho)(1-\beta\rho))} \quad \phi_{12} = \frac{2\theta}{a+\theta(1+\beta)+\Lambda}$$

$$\Lambda = \sqrt{(a+\theta(1-\beta))^2+4a\beta\theta} \quad \theta = \frac{1}{\mu a}$$

According to Figures 1 and 2, the performance of nominal income growth targeting vis-à-vis the other monetary policy strategies depends critically on the size of a and μ . Nominal income growth targeting replicates the behavior of the rate of inflation and the output gap under optimal policy from the timeless perspective provided that $\mu a = 1$. In this case, the target rules for both policies are identical. Except for small values of μ , nominal income growth targeting performs better than pure discretion and standard commitment when $a=0.01$. Indeed for $75 \leq \mu \leq 150$ its performance vis-à-vis the timeless perspective is only marginally worse with relative losses amounting to less than 3%. As a increases in size to 0.05 the attractiveness of nominal income growth targeting wanes quickly. For small values of μ its performance compared to the timeless perspective is still respectable, notably over the range of $10 \leq \mu \leq 40$ where relative losses amount to less than 10%. However, as μ continues to increase, losses under nominal income growth targeting rise rapidly, first exceeding those associated with standard commitment and eventually those associated with pure discretion. Around the $\mu = 200$ mark, nominal income growth targeting becomes the least attractive of the four strategies.

4. Conclusion

Taken altogether, from a theoretical perspective nominal income growth targeting can be a suitable choice for policy in an environment where price stability is the primary but not exclusive concern of policy. However, this positive assessment of the stabilizing properties of nominal income growth targeting depends largely on a rather small value for the key parameter a .

References

- Clarida, R., Jordi G., Gertler, M., 1999. The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature*, 27, 1661-1707.
- Jensen, H., 2002. Targeting Nominal Income Growth or Inflation? *American Economic Review*, 92 (4), 928-956.
- McCallum, B.T., Nelson, E., 2004. Timeless Perspective vs. Discretionary Monetary Policy in Forward-Looking Models. *Federal Reserve Bank of St. Louis Review*, 86 (2), 43-56.
- Walsh, C.E., 2003. The Output Gap and Optimal Monetary Policy. *American Economic Review*, 93 (1), 265-278.

Figure 1:

Figure2:

Note: $\sigma_u^2 = \frac{1}{1-\rho^2} \sigma_{\tilde{u}t}^2$ $\sigma_u^2 = 0.000009$ Scaling Factor: 100000