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**PAYING VS. WAITING IN THE PURSUIT OF SPECIFIC
EGALITARIANISM**

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PAYING VS. WAITING IN THE PURSUIT OF SPECIFIC EGALITARIANISM

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ABSTRACT. We propose an allocation mechanism for publicly providing a private good such that the final allocation is simultaneously independent of income and increasing in strength of preference or need. The “pay or wait” mechanism consists of offering the good for sale at two outlets. The ‘queuing’ outlet would charge a low money price per unit, but high waiting timer per unit. The ‘pricing’ outlet would charge a relatively high money price with rapid service. High wage individuals will opt for the pricing outlet, and low wage individuals the queuing outlet. If the policy maker stocks the outlets in proportion to the distribution of high and low wage earners in the population, consumers of both wages will purchase the same amount on average, while those who value the good more relative to other goods will receive more of it. These outcomes are at risk if the good can be privately resold, but may be preserved if the policy maker can create transactions costs associated with resale.

Keywords: in-kind provision, redistribution, specific egalitarianism

JEL Classifications: D30, D45, H31, I18

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1. Introduction

When it comes to society's concern over consumption inequality, some goods are more equal than others. As Tobin observed in 1970, our general acceptance of inequality is tempered for those commodities 'essential to life and citizenship,' such as early education, emergency health care, voting, government services, food in times of crisis, park or beach access, and so on. Traditionally, economists have been wary of distributional concerns over specific private goods ('paternalism'), and of the public provision of such goods for redistributive purposes. It was commonly argued that distributional concerns motivated by utilitarian social welfare could be met at least cost by redistributing *income* from rich to poor, and then allowing market prices to allocate resources to their most valued uses (the Second Welfare Theorem).

Nonetheless, two changes since 1970 have increased the attention economists have paid to public in-kind provision. First, good-specific distributional concerns have proved sufficiently robust over time that some economists suggest they be taken seriously as public policy objectives (Weitzman, 1977; Rosen, 2002, p. 175). For example, compulsory public health insurance in Canada, implemented federally in 1968, was reviewed in 2002 and justified in part on the basis that Canadians want the poor to have the same access to health care as the rich (Romanow Commission, 2002, p. xvi). Tobin (1970) was the first to characterize such non-welfarist objectives as *specific* egalitarianism. Rationing scarce health services by queues rather than price seems acceptable from this view, because time is more equally distributed than earnings ability or wealth (Nichols, Smolensky and Tideman, 1971).

A second change within economics leading to a focus on in-kind provision has been the incorporation of imperfect information into standard welfare economics. Goods like health insurance may not be available to all in private markets if providers

cannot distinguish high and low risk individuals. Government provision of uniform compulsory insurance may thus be welfare-improving (Rothschild and Stiglitz, 1976). Similarly, charities or governments may create a ‘samaritan’s dilemma’, or form of moral hazard where individuals correctly anticipate that if they under-invest in precautionary goods when young, they will be rescued from bad outcomes later in life. Thus people under-save and under-insure (Bruce and Waldman, 1991). Most importantly, governments may be unable to distinguish high from low ability workers, and thus face restrictions on the degree of redistribution possible through optimally designed tax systems. This is because individuals with high ability may mimic those with low ability in order to avoid taxes or qualify for cash transfers (Blackorby and Donaldson, 1988; Boadway and Marchand, 1995; and Blomquist and Christiansen, 1997). Unknown risk, moral hazard, and unknown ability have all been used to identify conditions under which social welfare could be higher if certain private goods were publicly provided at a uniform level to all.

Our paper draws on both of the above considerations to propose an in-kind redistributive mechanism that recognizes self-selection constraints for high and low ability individuals. The redistributive objective we consider is commodity-specific egalitarianism. Specific target goods such as elective health care, government services, secondary disaster relief, or access to national parks or campgrounds could be made available through parallel outlets that ration by different combinations of price and time. A policy maker, by choosing the allocation of the good and its money (or time) price across outlets, can ensure that individuals self-select outlets by their earnings ability. Those with a relatively high earnings ability will choose outlets that ration more by paying than by waiting, and vice versa. Under conditions we identify, our mechanism can achieve specific egalitarianism, ensuring that individuals with a

given strength of preference for the target good will purchase the same amount regardless of income, while those who value it more highly relative to other goods will purchase more than those who value it less.

Of course, any use of time as an allocation device involves the waste of an otherwise valuable resource. There is then inescapably an efficiency cost to the ‘pay or wait’ redistribution mechanism we propose. It is beyond the scope of this paper to compare the tradeoff between equity and efficiency achieved by this mechanism with that achieved by more traditional tax and transfer systems under imperfect information. Rather, we focus on the distributional properties that a ‘pay or wait’ mechanism can achieve, with and without the potential for private resale. We note heuristically how efficiency costs can be minimized, and refer interested readers to Clark and Kim (2005) for greater detail.¹

The layout of the paper is as follows. Section 2 provides a brief review of the literature on the distributional and efficiency aspects of queuing as a redistributive mechanism. Section 3 provides a formal model of our allocation mechanism, with and without the potential for resale, and with the introduction of preference heterogeneity. We conclude in Section 4.

2. Waiting for Godot

As noted by Nichols et al. (1971), Barzel (1974), O’Shaughnessy (2000), and Alexeev and Leitzel (2001), allocating goods that are deemed essential to citizenship or life using queues rather than price can seem appealing in an egalitarian sense, because time is more evenly distributed than human or physical capital, or income.

Economists, in contrast, have criticized queuing on two major efficiency grounds. First, buyers who wait in line are surrendering a valuable resource, time,

that unlike money does not get transferred to the seller. The opportunity cost of that time includes not just leisure, but forgone production. Thus, widespread queuing for goods in an economy would ultimately make fewer of these goods available. Secondly, the time price of queuing penalizes those with a higher opportunity cost of time. When compared to money pricing, queuing will thus transfer goods from some who value them more to others who value them less (Tobin, 1970; Suen, 1989; and O'Shaughnessy, 2000). Far better to meet distributional concerns at a general level with a tax and transfer system, and then allocate private goods by price and congestible public services with user fees set at marginal social cost.

As previously pointed out, however, imperfect information means that tax and transfer systems carry their own distortions in work disincentives (Tobin, 1970; Bucovetsky, 1984) and imperfect targeting (Alexeev and Leitzel, 2001). Similarly, user fees for congestible public services may have regressive distributional effects (Nichols et al., 1971). In response, a number of studies have compared the efficiency of alternative re-distributional instruments, such as tax/transfers, in-kind transfers, queuing, or rationing with resale (Bucovetsky, 1984; Sah, 1987; Blackorby and Donaldson, 1988; Polterovich, 1993; O'Shaughnessy, 2000; and Alexeev and Leitzel, 2001). In general, if re-sale is not practical, the inefficiency of queuing must be traded-off against the inefficiency of allocating uniform quantities of a good to heterogeneous people.

Alternatively, Nichols et al. (1971) had a key insight that if people could *choose* whether to pay by money or by time, much of the re-distributional potential of allocation by time could be preserved, and its inefficiency reduced. Indeed, private firms with a degree of monopoly power commonly sell goods using a menu of price / wait combinations as a form of second degree price discrimination to increase profits

(Tirole, 1988). Governments could do the same with a target good of interest, but to pursue distributional ends such as specific egalitarianism. Low wage individuals would self-select to pay by time, while those with a high wage would self-select to pay by money. If wage captures the opportunity cost of time, and differences in wages reflect differences in marginal product, then the time lost in queues would have low foregone cost in wages and production. The costly and error-prone apparatus of means testing individuals would be unnecessary.

While Nichols et al. (1971) provided no formal model of parallel markets, O'Shaughnessy (2000) and Alexeev and Leitzel (2001) have when comparing social welfare under such systems with that under conventional tax and transfer systems. Both of these studies consider general equilibrium production economies in which households value leisure and a single consumption good. Households may purchase the consumption good either at a subsidized price with a queue, or at a higher free market price without a queue. O'Shaughnessy (2000) finds that the tradeoff between mean *consumption* and its inequality may be better under differential pricing than a tax and welfare system. However, the tradeoff between mean *utility* and its inequality would favour the tax and welfare system. In contrast, Alexeev and Leitzel (2001) find that the tradeoff between mean utility and its inequality may favour parallel markets over lump sum taxes and transfers when the latter are imperfectly targeted. Interestingly, both papers find that parallel markets gain an advantage over tax and transfers as the social planner increases the weight on inequality relative to mean consumption (O'Shaughnessy) or income (Alexeev and Leitzel).

Our approach differs from O'Shaughnessy (2000) and Alexeev and Leitzel (2001) in that we ask whether parallel markets can achieve the particular distributional objective of specific egalitarianism, or that the consumption of a target

good be independent of income, but dependent on strength of preference. We thus divide their models' single consumption good into a target good, and a remaining composite commodity over which society has no distributional concerns. We also differ in allowing preference or need for the target good to vary across the population. Finally, we differ in allowing the policy maker the extra degree of freedom of setting a money price in *both* parallel outlets, and through this the ability to create or preclude queuing in the outlet targeted to the rich.

3. Our model

Consider an economy of N people who have preference orderings over leisure l , a composite commodity y , and a good g targeted by the government. We will assume that people's preferences can be represented by a constant elasticity of substitution (CES) utility function², and initially that they are identical:

$$U(l, y, G) = (l^\rho + y^\rho + \theta g^\rho)^{\frac{1}{\rho}}, \quad (1)$$

where θ represents an individual's strength of preference for the target good relative to the other goods. Rho (ρ) is a positive monotonic transformation of the person's elasticity of substitution between the target and other goods, ranging from perfect flexibility ($\rho = 1$), to Cobb Douglas ($\rho = 0$), to Leontief ($\rho = -\infty$).³ As we shall see, the value of rho plays a key role in determining how differences in income affect people's time allocation decisions.

The price of leisure is a person's wage, w , while the price of y is normalized to 1. The *full* money and time price of the target good at a given outlet is $P_g = wH + p$, where p is the money unit price, and H is the waiting time required per unit of g purchased.⁴ We assume initially that g cannot feasibly be resold once purchased. All

individuals have an identical time endowment, T , which they can spend working L , in leisure ℓ , or in line (Hg) . We assume that all income comes from labour, but do not explicitly model the allocation of labour hours to the production of y and g . Instead, we take the supply of g as exogenous at M .⁵

For ease of exposition, we shall start by assuming that g is available from only one outlet, with no potential for resale, and that all individuals face the same wage w .

An individual's problem is:

$$\begin{aligned} \text{Max}_{L, l, y, g} U &= (1^\rho + y^\rho + \theta g^\rho)^{\frac{1}{\rho}} \\ \text{s.t.} \quad &wL = pg + y, \\ &T = 1 + L + Hg, \\ &L \geq 0. \end{aligned} \tag{2}$$

With interior solutions for any $\rho \in (-\infty, 1)$, the corresponding supply and demand functions are:

$$\begin{aligned} L^* &= \left(\frac{(\theta^{\frac{1}{1-\rho}} P_g^{\frac{1}{\rho-1}} p + 1)T}{D} \right), \\ l^* &= \left(\frac{w^{\frac{\rho}{\rho-1}} T}{D} \right), \\ y^* &= \left(\frac{wT}{D} \right), \text{ and} \\ g^* &= \left(\frac{w P_g^{\frac{1}{\rho-1}} \theta^{\frac{1}{1-\rho}} T}{D} \right), \end{aligned} \tag{3}$$

where $D = w^{\frac{\rho}{\rho-1}} + P_g^{\frac{\rho}{\rho-1}} \theta^{\frac{1}{1-\rho}} + 1$.

Consider now the outlet for the target good, with a fixed supply of the target good, M . The technical conditions are satisfied to ensure that there exists a unique full price P_g^* that clears the outlet:⁶

$$Ng^* = M \quad (4)$$

Note that since it is the *full* price $P_g^* = wH^* + p^*$ which clears the outlet, the policy maker has freedom to set a negative, zero, or positive money price $p^* \leq P_g^*$, and let the per unit queuing time H^* (≥ 0) adjust to satisfy (4). Alternatively, the policy maker can set a target non-negative per unit queuing time H^* and let the money price p^* adjust to satisfy (4). We shall primarily frame our discussion, however, in terms of money price setting.

3.1 The isoprice line

The policy maker could set the money price just high enough to enable the market for the target good to clear without any queue, or $P_g^* = p_p^*$. We label this ‘pure pricing.’ In contrast, the policy maker could set the money price at zero, causing the market to clear by ‘pure queuing’ at $P_g^* = wH_q^*$. Between these extremes, the money price could be set at a positive intermediate level $0 < p^* < p_p^*$, resulting in an intermediate equilibrium queuing time $0 < H^* < H_q^*$ and full price $P_g^* = wH^* + p^*$. Finally, the money price could be set at a negative level as a unit subsidy, $p^* < 0$, requiring even greater equilibrium queuing time than under pure queuing. In equilibrium, this negative money price would be constrained at the p_{min} below which each individual would if possible supply negative labour hours and ‘purchase g for a living.’ From (3), $L^* = 0$ when $p_{min}^* = -\theta^{\frac{1}{\rho-1}}(P_g^*)^{\frac{1}{1-\rho}}$, which would have a

corresponding maximum time price $H_{\max}^* = \frac{(-p_{\min}^*)^{1-\rho} \theta - p_{\min}^*}{w}$. Following Nichols et al. (1971), let us mark these various equilibrium money/time price pairs in (H, p) space in Fig.1.

With wage uniform at w , and p chosen by the policy maker, a unique value of H^* will determine the P_g^* that satisfies (4) and clears the market. For an individual with wage w , these (H^*, p^*) combinations form an isoprice line $p^* = P_g^* - wH^*$ with slope $-w$. This person would pay the same full price at any point on the isoprice line, a higher full price at (H, p) combinations above it, and a lower full price at (H, p) combinations below it.

To introduce income inequality, we first examine the effect of an increase in the (uniform) wage on the position of the isoprice line for a person at a sole outlet. Consider first the intercepts. At the pure pricing intercept $P_g^* = p_p^*$, we differentiate (4) with respect to w . This yields

$$\frac{\partial p_p^*}{\partial w} = \frac{\left(\frac{\rho}{\rho-1} \right) w^{\frac{\rho}{\rho-1}} - D}{w \left[\frac{1}{\rho-1} p_p^{*-1} \left(w^{\frac{\rho}{\rho-1}} + 1 \right) - \theta^{\frac{1}{1-\rho}} p_p^{*\frac{1}{\rho-1}} \right]}. \quad (5)$$

Examining (5), we see that $\frac{\partial p_p^*}{\partial w} > 0$ for all values of $\rho \in (-\infty, 1)$. Thus, under pure pricing, a wage increase for all individuals would cause demand for g to shift right, raising the money price needed to clear the market.

At the pure queuing intercept $P_g^* = wH_q^*$, differentiating (4) yields

$$\frac{\partial H_q^*}{\partial w} = \frac{-\rho w^{\frac{2\rho-1}{1-\rho}}}{(1-\rho)H_q^{*\frac{1}{\rho-1}}\theta^{\frac{1}{1-\rho}} + H_q^{*-1}(1+w^{\frac{\rho}{1-\rho}})}. \quad (6)$$

Fig. 1. A market-clearing isoprice line for a single outlet and wage group

Examining (6), we see that $\frac{\partial H_q^*}{\partial w} < 0$ for $0 < \rho < 1$, $\frac{\partial H_q^*}{\partial w} = 0$ for $\rho = 0$, and

$\frac{\partial H_q^*}{\partial w} > 0$ for $\rho < 0$. That is, if the outlet clears only by queuing, a higher wage would

lower the equilibrium time price if the elasticity of substitution between goods were high ($\rho > 0$), but raise it if that elasticity were low ($\rho < 0$). Intuitively, this is because a higher wage has an income effect that makes people demand more of all goods, including the target good. But a higher wage also raises the opportunity cost of the target good when purchased only with time (as it does leisure), creating a substitution effect towards labour and y and away from demand for the target good or leisure. The income effect of a wage increase dominates the substitution effect when $\rho < 0$, causing net demand for the target good to rise, and the equilibrating time price to rise. The two effects exactly offset when $\rho = 0$, leaving net demand and equilibrating time price unchanged. Finally, the substitution effect of a wage increase dominates the

income effect when $\rho > 0$, causing net demand for the target good to fall, and the equilibrating time price to fall.

3.2 Two outlets without resale

Returning to our isoprice line in Fig.1, we can now predict the effect of raising everyone's wage from w (redefined as w_L) to w_H . For $\rho > 0$, the signs of the partial derivatives in (5) and (6) and the steeper slope imply that an individual's isoprice line of market-clearing (H, p) combinations will rotate clockwise around a point (H^{**}, p^{**}) in the positive quadrant, as shown in Fig. 2. For $\rho < 0$, the signs of (5) and (6) and the steeper slope imply that the isoprice line will rotate clockwise around a point (H^{**}, p^{**}) in the negative quadrant, as shown in Fig. 3. More precisely, the new isoprice line will cross the original so long as its lower bound $(H_{H,max}^*, p_{H,min}^*)$ lies to the southwest of $(H_{L,max}^*, p_{L,min}^*)$. This will indeed occur, as can be seen by returning to the definitions of H_{max}^* and p_{min}^* at zero labour supply. If p_{min}^* is held constant at its negative value,

$$\frac{\partial H_{max}^*}{\partial w} = - \frac{[(-p_{min}^*)^{1-\rho} \theta - p_{min}^*]}{w^2} < 0. \quad (7)$$

Similarly, if H_{max}^* is held constant,

$$\frac{\partial p_{min}^*}{\partial w} = \frac{[-\theta^{\frac{1}{\rho-1}} \frac{1}{1-\rho} (P_g^*)^{\frac{\rho}{1-\rho}}] H_{max}^*}{[1 + \theta^{\frac{1}{\rho-1}} \frac{1}{1-\rho} (P_g^*)^{\frac{\rho}{1-\rho}}]} < 0. \quad (8)$$

With a slight change of interpretation, we can use Figs 2 or 3 to introduce multiple income groups and outlet choice to our model. Suppose that there are two income groups, with N_L individuals earning a wage w_L , and $N_H = N - N_L$ individuals earning a wage w_H , where $w_H > w_L$.⁷ To achieve equal consumption of g , the policy

Fig. 2. Isoprice lines for two wage levels, $\rho > 0$

Fig. 3. Isoprice lines for two wage levels, $\rho < 0$

maker could create two outlets where the target good is available. For the moment, we continue to assume that resale is not possible.

At a ‘queuing’ outlet, the policy maker would set a lower money price, p_L , to attract the poor, while at a ‘pricing’ outlet he would set a higher money price, p_H , to attract the rich. The policy maker must distribute the supply of the target good M across the outlets, M_q and M_p , in proportion to the income distribution:

$$M_q = \frac{N_L}{N} M \quad \text{and} \quad M_p = \frac{N_H}{N} M . \quad (9)$$

If the poor and rich self-select to their respective outlets, each outlet will clear according to its own version of (4). With supply at each outlet pre-set in proportion to distribution of earner types as in (9), the isoprice lines of rich and poor at each outlet will differ only in wage. Their isoprice lines will then cross under the same conditions as in our thought experiment for a single outlet in Figs 2 or 3.

At the queuing outlet, the policy maker is free to set the money price p_L anywhere between $p_{L,\min}^*$ and p^{**} . For example, it could be set at zero when $\rho > 0$, resulting in the outlet clearing by pure queuing, or formally at the H_{qL}^* where

$$N_L g_L^* = N_L \left(\frac{w_L (w_L H_{qL}^*)^{\frac{1}{\rho-1}} \theta^{\frac{1}{1-\rho}} T}{w_L^{\frac{\rho}{\rho-1}} + (w_L H_{qL}^*)^{\frac{\rho}{\rho-1}} \theta^{\frac{1}{1-\rho}} + 1} \right) = M_q . \quad (10)$$

At maximum, p_L could be set at the unique (H^{**}, p^{**}) combination that would simultaneously clear a proportionately stocked queuing outlet with poor people or a proportionately stocked pricing outlet with rich people,

$$N_L g_L^* = N_L \left(\frac{w_L (w_L H^{**} + p^{**})^{\frac{1}{\rho-1}} \theta^{\frac{1}{1-\rho}} T}{w_L^{\frac{\rho}{\rho-1}} + (w_L H^{**} + p^{**})^{\frac{\rho}{\rho-1}} \theta^{\frac{1}{1-\rho}} + 1} \right) = M_q \quad (11)$$

$$N_H g_H^* = N_H \left(\frac{w_H (w_H H^{**} + p^{**})^{\frac{1}{\rho-1}} \theta^{\frac{1}{1-\rho}} T}{w_H^{\frac{\rho}{\rho-1}} + (w_H H^{**} + p^{**})^{\frac{\rho}{\rho-1}} \theta^{\frac{1}{1-\rho}} + 1} \right) = M_p .$$

The policy maker has similar flexibility in setting the money price at the pricing outlet. At maximum, p_H could be set to clear the outlet without queuing when only rich people are in it, or the p_{pH}^* where

$$N_H \mathcal{G}_H^* = N_H \left(\frac{w_H p_{pH}^* \frac{1}{\rho-1} \theta^{\frac{1}{1-\rho}} T}{w_H^{\frac{\rho}{\rho-1}} + p_{pH}^* \frac{\rho}{\rho-1} \theta^{\frac{1}{1-\rho}} + 1} \right) = M_p. \quad (12)$$

At minimum p_H could be set at the p^{**} already defined in (11).

Before presenting our results, we describe finally how individuals choose between the queuing and pricing outlets to purchase the target good. An individual will choose the outlet that offers the lowest *full* price given his wage. With CES utility, the composition of that full price will affect the individual's time allocation between work and queuing, but not his demand for leisure. More formally, the individual's demand functions given in (3) are conditional on his choice of outlet. Substituting these into utility yields indirect utility

$$V = \left((1^*)^\rho + (y^*)^\rho + \theta (g^*)^\rho \right)^{\frac{1}{\rho}} = \left(1 + P_g^* \frac{\rho}{\rho-1} \theta^{\frac{1}{1-\rho}} w^{\frac{\rho}{1-\rho}} + w^{\frac{\rho}{1-\rho}} \right)^{\frac{1-\rho}{\rho}} T, \text{ where } P_g^* = wH^* + p^*. \quad (13)$$

It follows that for any $\rho \in (-\infty, 1)$, $V \Big|_{\hat{p}_g^*} > V \Big|_{\hat{p}_g^*}$ if and only if $\hat{P}_g^* < \hat{P}_g^*$. We now have sufficient background to present our results.

Proposition 1 Suppose a society has two income levels, and a target good for which resale is not possible. If a policy maker creates two outlets, distributes the target good proportionally as in (9), and chooses money prices $(p_{L,\min}^* \leq p_L \leq p^{**})$ and $(p^{**} \leq p_H \leq p_{pH}^*)$ such that $p_H \neq p_L$, he will induce a unique separating equilibrium where the rich choose the pricing outlet, and the poor choose the queuing outlet.

Proof See Appendix 1

The intuition for Proposition 1 can be grasped easily from Fig. 2 or Fig. 3. Suppose the policy maker chooses a p_L for the queuing outlet that corresponds to point A on a poor person's isoprice line, and a p_H for the pricing outlet that corresponds to point B on a rich person's isoprice line (in either Figure). This will create a separating equilibrium, because a poor person who switched from A to B would face an (H, p) combination that posed a higher full price given his low wage. Conversely, a rich person who switched from B to A would face an (H, p) combination that would pose a higher full price given his high opportunity cost of time.

Once we have established that the rich and poor separate, it is easy to show that they purchase the same quantity of the target good.

Proposition 2 The separating equilibrium above equalizes consumption of the target good between rich and poor.

Proof With money prices set by the policy maker, the queuing time at the pricing and queuing outlets adjust such that

$$N_L g_L^* = M_q \text{ and } N_H g_H^* = M_p. \quad (14)$$

From proportional allocation as in (9), it follows that

$$g_L^* = M_q \frac{1}{N_L} = \left(\frac{N_L}{N} M\right) \frac{1}{N_L} = \frac{M}{N}, \text{ and} \quad (15)$$

$$g_H^* = M_p \frac{1}{N_H} = \left(\frac{N_H}{N} M\right) \frac{1}{N_H} = \frac{M}{N}.$$

Propositions 1 and 2 show that a policy maker can accommodate two wage groups at two outlets with flexibility in setting money prices. Note that the degree of the policy

maker's flexibility depends on the extent of the wage disparity between the two groups. Holding w_L constant, an increase in w_H would rotate the high wage isoprice line clockwise, either in the positive ($\rho > 0$) or negative ($\rho < 0$) quadrants. This in turn would decrease H^{**} and raise p^{**} , and increase the range of money prices that the policy maker could choose for both income groups while preserving separation.⁹ In particular, a decrease in H^{**} would allow the policy maker to set p_L so as to lessen the time the poor must spend in queues.

Of course, queuing time and its associated waste will be minimized if both outlet's money prices are set at their maximum possible values, p^{**} and p_{pH}^* . On the other hand, if the policy maker is willing to tolerate having both income groups queue, it turns out that he can achieve the same equal allocation of g using one outlet rather than two. To do so, he must set the single money price at the unique level identified in (11).

Proposition 3 There exists a pooled outlet with a unique (H^{**}, p^{**}) at which both income groups face the same full price and consumption of g as at the separating equilibrium.

Proof: if the proportional allocation of the target good is combined in a single outlet that charges (H^{**}, p^{**}) , a low income person will face the same full price at the pooled outlet as he did at the separating one ($P_g^*|_{w_L} \equiv P_{g,L}^* = w_L H^{**} + p^{**} = w_L H_L^* + p_L^*$). This means he will be indifferent between the two equilibria, and still demand $g_L^* = M/N$. The high income person will also face the same full price at the pooled and separating outlets

$(P_g^*|_{w_H} \equiv P_{g,H}^* = w_H H^{**} + p^{**} = w_H H_H^* + p_H^*)$, be indifferent between them, and demand $g_H^* = M/N$. The pooled outlet will clear, as

$$N_L g_L^* + N_H g_H^* = N_L (M/N) + N_H (M/N) = M. \quad (16)$$

To summarize, we have shown that when resale of a target good is not feasible, a policy maker can make its consumption independent of income by inducing different money and time prices for different income groups. We next examine the consequences for this mechanism's distributional outcome if the good can be resold.

3.3 Two outlets with resale

A potential problem with differential pricing emerges if the mechanism is applied to a good that can be resold. As illustrated by the vertical intercepts of the isoprice lines in Figs. 2 or 3, the (pure pricing equivalent) full price of g at the separating equilibrium without resale is higher for the rich, $P_g^*|_{w_H}$ than for the poor, $P_g^*|_{w_L}$. If resale were feasible, such price differences would provide high wage individuals the incentive to bypass the pricing outlet, and offer an intermediate price p_b to low wage individuals to resell the good.¹⁰ Taking some g purchased from the queuing outlet and reselling it to the rich could provide the poor with a higher utility than retaining it for own consumption, making both parties to the transaction better off. However specific egalitarianism would no longer be achieved, as final consumption of g would be higher among the rich than the poor. We pose the problem more formally below, adapting the resale model of Stahl and Alexeev (1985), and propose an approach the policy maker could take to preserve specific egalitarianism when resale is feasible.

To characterize resale market equilibrium, we first define $g_{b,i}$ as the quantity of the target good that is traded by an individual of wage i in a resale market. For low wage individuals who sell in the resale market, $g_{b,L} \leq 0$. For high wage individuals who buy in the resale market, $g_{b,H} \geq 0$. $(g_L - g_{b,L})$ is then defined as the total quantity of the target good purchased by a low wage individual at the queuing outlet, while $(g_H - g_{b,H})$ is the quantity of the target good purchased by a high wage individual at the pricing outlet. Reselling units of the target good at p_b provides the poor with an effective hourly wage of $w_b \equiv \frac{p_b - p_L}{H_L}$. While the policy maker may pre-commit to a money or time price at each outlet, we assume for convenience that he sets the latter at $(H^{**} \leq H_L^* < H_{L,\max}, H_H^* = 0)$, and allows the money prices to equilibrate.¹¹ An individual's problem, conditional on choice of outlet, becomes

$$\begin{aligned} \text{Max}_{L,1,y,g} U &= \left(1_i^\rho + y_i^\rho + \theta g_i^\rho\right)^{\frac{1}{\rho}} \\ \text{s.t.} \quad w_i L_i - p_b g_{b,i} &= p_i (g_i - g_{b,i}) + y_i, \\ T_i &= 1_i + L_i + H_i (g_i - g_{b,i}), \\ L_i \geq 0, (g_i - g_{b,i}) &\geq 0, \quad i = L, H. \end{aligned} \quad (17)$$

For high wage individuals, the corresponding supply and demand functions are:

$$\ell_H^o = \left(\frac{(\theta^{\frac{1}{1-\rho}} P_{g,H}^{\frac{1}{\rho-1}} p_H + 1)T}{D_H} \right), \quad y_H^o = \left(\frac{w_H^{\frac{\rho}{\rho-1}} T}{D_H} \right), \quad y_H = \left(\frac{w_H T}{D_H} \right), \quad g_H = \left(\frac{w_H \theta^{\frac{1}{1-\rho}} P_{g,H}^{\frac{1}{\rho-1}} T}{D_H} \right), \quad (18)$$

where $D_H = 1 + w_H^{\frac{\rho}{\rho-1}} + P_{g,H}^{\frac{\rho}{\rho-1}} \theta^{\frac{1}{1-\rho}}$, and $P_{g,H} = p_H$. Note that p_H may differ from the case without resale. For low wage individuals, the corresponding supply and demand functions are:

$$\ell_L^o = 0, \quad \ell_L^o = \left(\frac{w_b w_L^{\frac{1}{\rho-1}} T}{D_{b,L}} \right), \quad \ell_p^o = \left(\frac{w_b T}{D_{b,L}} \right), \quad g_p^o = \left(\frac{w_b \theta^{\frac{1}{1-\rho}} P_{g,L}^{\frac{1}{\rho-1}} T}{D_{b,L}} \right), \quad (19)$$

where $D_{b,L} = 1 + w_b w_L^{\frac{1}{\rho-1}} + p_b P_{g,L}^{\frac{1}{\rho-1}} \theta^{\frac{1}{1-\rho}}$, and $P_{g,L} = p_L + w_L H_L$. Note that the introduction of an effective wage $w_b > w_L$ induces low wage individuals to supply no labour in their prior jobs.

Equilibrium here requires that the pricing, queuing and resale markets clear simultaneously, or that

$$N_L (g_p^o - g_{p,L}^o) = M_q, \quad (20)$$

$$N_H (g_p^o - g_{p,H}^o) = M_p, \quad (21)$$

$$N_L g_{p,L}^o + N_H g_{p,H}^o = 0. \quad (22)$$

As we show below, there exist a unique set of prices that satisfy equations (20)-(22) such that $P_g^*|_{w_H} > \ell_g^o|_{w_H} = \ell_p^o = \ell_g^o|_{w_b} > P_g^*|_{w_L}$. This single resale market price ℓ_p^o eliminates arbitrage across markets, but compared to the case without resale, results in a lower full price for the rich and higher full price for the poor.

More formally, the resale price ℓ_p^o cannot be less than the price at the queuing outlet $\ell_g^o|_{w_b} \equiv \ell_p^o + w_b H_L^o$ or the poor would purchase nothing there $(g_p^o - g_{p,L}^o) = 0$ and (20) would be violated. The resale price ℓ_p^o similarly cannot be less than the price at the pricing outlet $\ell_g^o|_{w_H} \equiv \ell_p^o + w_H H_H^o$ or the rich would purchase nothing there $(g_p^o - g_{p,H}^o) = 0$ and (21) would be violated. Conversely, the resale price ℓ_p^o cannot exceed the price at the queuing outlet, $\ell_g^o|_{w_b}$. Low wage individuals would then seek to sell infinite quantities into the resale market. Their time constraints would prevent

this, exerting downward pressure on w_b via the resale price until $\beta_\theta = \beta_g^0|_{w_b}$. Similarly, the resale price β_θ cannot exceed the price at the pricing outlet $\beta_g^0|_{w_H}$ or else the rich would not patronize the resale market, and $\theta_{\theta,H} = 0$. Yet if β_θ is greater than $P_g^*|_{w_H}$, which in turn is greater than $P_g^*|_{w_L}$, then the poor's effective wage $w_b (\equiv \frac{\beta_\theta - P_L^*}{H_L^*})$ must exceed w_L , giving the poor an incentive to sell into the resale market ($\theta_{\theta,L} < 0$). This would violate (22). We thus have $\beta_g^0|_{w_H} = \beta_\theta = \beta_g^0|_{w_b}$.

To compare the equilibrium resale price to its level when resale was not possible, we assume that the policy maker fixes H_L^* at the same level with resale that they would have without resale. We argue first that the equilibrium resale price β_θ cannot be less than the equilibrium queuing outlet price that occurred without resale, $P_g^*|_{w_L}$, or else the poor would not patronize the queuing outlet, $(\theta_L - \theta_{\theta,L}) = 0$, and (20) would be violated. β_θ also cannot equal $P_g^*|_{w_L}$, for then the poor would have no incentive to resell, and $\theta_{\theta,L} = 0$. Yet if β_θ is equal to $P_g^*|_{w_L}$, which in turn is less than $P_g^*|_{w_H}$, then the rich would want to purchase more in total than without resale, or $\theta_H > M/N$. Given the unchanging supply in the pricing outlet, M_p , this can only be achieved if $\theta_{\theta,H} > 0$, which would violate (22). Thus $\beta_\theta > P_g^*|_{w_L}$. Finally, we argue that the equilibrium resale price β_θ cannot equal or exceed the equilibrium pricing outlet price without resale, $P_g^*|_{w_H}$, or else the rich would have no incentive to patronize the resale market and $\theta_{\theta,H} = 0$. With the poor selling into the resale market

$(g_{\beta,L} < 0)$, this would violate (22). Together this yields our results

$$P_g^*|_{w_H} > \beta_g|_{w_H} = \beta_g = \beta_g|_{w_b} > P_g^*|_{w_L}.$$

The existence and uniqueness of these prices can be demonstrated as follows. If β_g were as low as $P_g^*|_{w_L}$, there would be excess demand for g in the resale market. In particular, high wage individuals would demand more than without resale (M/N), since $P_g^*|_{w_L} < P_g^*|_{w_H}$. Low wage individuals will demand for themselves the same quantity as under resale (M/N), and will have no incentive to purchase extra for resale. Thus total demand for g will exceed supply, or $N_L g_{\beta,L} + N_H g_{\beta,H} > M_q + M_p = M$. By comparing this to the addition of equations (20) and (21), it can be shown that $N_L g_{\beta,L} + N_H g_{\beta,H} > 0$. Conversely, if β_g were as high as $P_g^*|_{w_H}$, there would be excess supply for g in the resale market. For high wage individuals would demand $g_{\beta,H} = M/N$ as without resale, either using the resale market $g_{\beta,H} > 0$ or not, $g_{\beta,H} = 0$. Positive purchase from the resale outlet $g_{\beta,H} > 0$ would imply that $(g_{\beta,H} - g_{\beta,H}) < M/N = M_p/N_H$, which would violate (21). Zero purchase, $g_{\beta,H} = 0$, together with the positive supply in the resale market ($g_{\beta,L} < 0$) shown in the paragraph above for $\beta_g = P_g^*|_{w_H}$, would imply $N_L g_{\beta,L} + N_H g_{\beta,H} < 0$. The uniqueness of β_g follows from monotonicity of the demand functions for g .

It can be shown that at the equilibrium when resale is feasible, both rich and poor would prefer the market equilibrium with resale to the former allocation without resale. For the poor, substituting the demand functions of (19) into the utility function will yield indirect utility V_L^0 . It can be shown that $V_L^0 > V_L^*$ if and only if

$$(w_L - \frac{(\beta_\theta - p_L^*)}{H_L^*}) H_L^* g_{\theta,L} \bar{D}_{\theta,L}^0 > 0, \quad (23)$$

where $\bar{D}_{\theta,L}^0$ is as defined in (19). The second expression in the parentheses in (23) is the effective wage from resale $\bar{w}_\theta$. This effective wage will exceed w_L , since

$$w_L = \frac{P_g^*|_{w_L} - p_L^*}{H_L^*}, \quad P_g^*|_{w_L} < \beta_\theta, \text{ and } H_L^* \text{ and } p_L^* \text{ are unchanged. As } g_{\theta,L} \text{ is negative, the}$$

combined product in (23) is positive. Thus a low wage individual would prefer the equilibrium with resale. For a high wage individual, substituting the demand functions of (18) into utility will yield V_H^0 . It can be shown that $V_H^0 > V_H^*$ if and only if

$$\beta_\theta = P_g^0|_{w_H} < P_g^*|_{w_H} = p_H^*. \quad (24)$$

This inequality was seen to hold above.

Since demand for the target good is falling in full price, it also follows that a given high wage individual will consume more at the market equilibrium with resale than at the former allocation, or $g_\theta > g_H^*$. A given low wage individual, facing a higher opportunity cost of retaining the target good, will consume less, or $g_\theta < g_L^*$. It follows that the poor will consume less of the target good than the rich, or $g_\theta > g_H^* = g_L^* > g_\theta$.

To preserve equal consumption of the target good when resale is feasible, we propose that the policy maker could structure sales so as to deliberately create transactions costs for either buyers or sellers who enter the resale market. For example, managers of campsites at popular national parks could mimic airlines by personalizing campsite passes with purchaser details. These details could then be verified using ID when the passes are presented for use. Those trying to use re-sold

passes must then forge or borrow ID, which requires additional time and money expense.

More formally, we denote the per unit money and time transactions costs created for low wage sellers or high wage buyers as $f_{m,i}$ and $f_{t,i}$, respectively, where $i = L, H$. We assume these costs depend linearly on the volume of g resold, though they could easily be modelled as independent of (positive) volume also. The total per unit transactions costs for poor and rich in resale are then $\alpha_L = f_{mL} + w_L f_{tL}$ and $\alpha_H = f_{mH} + w_H f_{tH}$, respectively.

The budget and time constraints of a low wage individual facing the option of resale with transactions costs are

$$w_L L_L - p_b g_{b,L} = p_L (g_L - g_{b,L}) - f_{m,L} g_{b,L} + y_L, \quad (25)$$

$$T_L = 1_L + L_L + H_L (g_L - g_{b,L}) - f_{t,L} g_{b,L}.$$

Substituting the low wage individual's resulting demand functions into his utility function, it can be shown that $V_L^0 > V_L^*$ if and only if

$$(w_L - \frac{(p_b^* - p_L^* - f_{m,L})}{H_L^* + f_{t,L}})(H_L^* + f_{t,L}) g_{b,L}^0 \bar{B}_{b,L}^0 > 0, \quad (26)$$

where $\bar{B}_{b,L}^0$ is defined as in (19). This inequality will hold if and only if

$$w_L < \frac{(p_b^* - p_L^*)}{H_L^*} - \frac{\alpha_L}{H_L^*}. \quad (27)$$

The policy maker could try to impose sufficient total per unit transactions costs $\alpha_{L,\min}$ on low wage individuals seeking to resell units of the target good that the inequality in (27) would not hold. Alternatively, if the transactions costs were imposed upon high wage individuals who could purchase g in the resale market, they would face budget and time constraints of

$$w_H L_H - p_b g_{b,H} = p_H (g_H - g_{b,H}) + f_{m,H} g_{b,H} + y_H, \quad (28)$$

$$T_H = 1_H + L_H + f_{t,L} g_{b,L},$$

or a combined constraint of

$$p_H (g_H - g_{b,H}) + (p_b + \alpha_H) g_{b,H} + y_H \leq w_H (T - 1_H). \quad (29)$$

As can be seen by examining (29), a high wage individual will be better off purchasing at least some g in the resale market, or $V_H^0 > V_H^*$ if and only if

$$p_b + \alpha_H < p_H. \quad (30)$$

Again the policy maker could try to impose sufficient total per unit transactions costs $\alpha_{H,\min}$ on high wage individuals seeking to purchase units of the target good from the resale market that the inequality in (30) would not hold. In either case, specific egalitarianism can be preserved if the policy maker imposes sufficient transactions costs on those attempting to buy or sell the target good in a resale market.

Moving from the problem of resale to that of preference heterogeneity, we ask whether our mechanism can ensure that consumption will depend on relative strength of preference, even as it equalizes consumption across wage groups. For simplicity of exposition, we return to the assumption that resale is infeasible.

3.4 Heterogeneous preferences

People vary in their willingness-to-pay for a good because they differ in income, but also because they differ in relative strength of preference or need. Our mechanism can respect the latter difference, rather than imposing an equal distribution of the good to all. That is, we ask our mechanism to make consumption of the target good independent of income, but increasing in strength of preference.¹²

We introduce heterogeneous preferences by allowing that individuals may place a ‘regular’ θ_R or a ‘strong’ weight θ_S on the target good in utility in equation (1),

where $0 < \theta_R < \theta_S$. Of N_L low wage individuals, a proportion s_L have θ_S weights, and $(1 - s_L)$ have θ_R weights. Of N_H high wage individuals, s_H have θ_S weights, and $(1 - s_H)$ have θ_R weights.

As we showed in (13), individuals with any given strength of preference for the target good will choose between outlets based only on full price P_g^* . It follows that individuals with differing θ 's but with an identical wage will choose the same outlet. Their isoprice lines will be identical in equilibrium, except that the lower bound for someone with high preference strength θ_S , $(H_{S,\max}^*, p_{S,\min}^*)$ will lie north-west of the lower bound for someone with θ_R , $(H_{R,\max}^*, p_{R,\min}^*)$.¹³ An example is provided in Fig. 5 below, where the dotted segment at the bottom of the high wage isoprice line represents money and time price combinations that would be feasible for someone with θ_R ($L_{H,R}^* \geq 0$), but infeasible for someone with θ_S ($L_{H,S}^* < 0$).

Note that with heterogeneity, the distribution of preferences among the people at an outlet will affect the full price that brings the outlet into equilibrium. For example, if only low wage individuals inhabit an outlet with a supply of M_q , there will exist a unique P_g^* that equates supply and demand:

$$(1 - s_L)N_L g_{L,R}^* + s_L N_L g_{L,S}^* = (1 - s_L)N_L \left(\frac{w_L P_g^{*\frac{1}{\rho-1}} \theta_R^{\frac{1}{1-\rho}} T}{w_L^{\frac{\rho}{\rho-1}} + P_g^{*\frac{\rho}{\rho-1}} \theta_R^{\frac{1}{1-\rho}} + 1} \right) + s_L N_L \left(\frac{w_L P_g^{*\frac{1}{\rho-1}} \theta_S^{\frac{1}{1-\rho}} T}{w_L^{\frac{\rho}{\rho-1}} + P_g^{*\frac{\rho}{\rho-1}} \theta_S^{\frac{1}{1-\rho}} + 1} \right) = M_q. \quad (31)$$

Differentiating (31), it can be shown that $\partial P_g^* / \partial s_L > 0$. Intuitively, an increase in the proportion of individuals with a strong preference for the target good at an outlet will bid up the (H, p) combinations that clear it. This would shift out the isoprice line of an individual at that outlet, regardless of his preference strength.

Let us return to the two outlet case from our previous discussion. Would the high and low wage groups separate as before into ‘pricing’ and ‘queuing’ outlets offering high p_H and low p_L money prices? Graphically, the isoprice lines for rich and poor must cross at an (H^{**}, p^{**}) combination that is feasible for all wage and preference types for a separating equilibrium to exist. That is, the isoprice lines must cross at an H^{**} between $(0, \min\{H_{L,S\max}^*, H_{L,R\max}^*, H_{H,S\max}^*, H_{H,R\max}^*\})$ so that the demand functions for the target good simultaneously satisfy

$$\begin{aligned} (1-s_L)N_L g_{L,R}^* + s_L N_L g_{L,S}^* &= M_q \quad \text{and} \\ (1-s_H)N_H g_{H,R}^* + s_H N_H g_{H,S}^* &= M_p. \end{aligned} \tag{32}$$

At one extreme, separating equilibria would not exist if the proportion of the poor with a strong taste for g , s_L , were too *high* relative to the analogous proportion of the rich, s_H . For a given s_H , the proportion s_L cannot exceed an upper bound of $s_{L\max}$ for H^{**} to remain nonnegative, as illustrated in Fig. 4. Intuitively, the greater the proportion of strong preference people at an outlet, the greater its full price, making other outlets with more tepid patrons more attractive. Thus, if s_L were too high at the queuing outlet relative to s_H at the pricing outlet, the isoprice lines would not intersect, and a marginal low wage person (of either preference strength) would prefer to switch to the pricing outlet.

At the other extreme, separating equilibria would not exist if s_L were too *low* relative to s_H , or less than the lower bound $s_{L\min}$. This is because a sufficient fall in s_L would shift the low wage isoprice line sufficiently inward to prevent the intersection of a low wage, high preference isoprice line with the high wage, high preference isoprice line. A marginal high wage person would then prefer to switch to the queuing outlet. Assuming neither extreme occurs, we may present our results.

Fig. 4. Maximum feasible range of s_L given s_H

Proposition 4 Suppose a society has two wage levels and two relative strengths of preference for g , such that $s_{L, \min} \leq s_L \leq s_{L, \max}$ for a given s_H . If a policy maker creates two outlets, distributes the target good as in (9), and chooses money prices $(p_{L, \min}^* \leq p_L \leq p^{**})$ and $(p^{**} \leq p_H \leq p_{pH}^*)$ where $p_H \neq p_L$, he will induce a unique separating equilibrium where the rich choose the pricing outlet, and the poor choose the queuing outlet.

Proof: so long as $(s_{L, \min} \leq s_L \leq s_{L, \max})$ given s_H , see Appendix 1 as before.

We turn next to ask whether this separating equilibrium will make consumption of g independent of income, but increasing in strength of preference. The queuing outlet will clear at the full price $(w_L H_L^* + p_L^*)$ that satisfies

$$(1-s_L)N_L \left(\frac{w_L(w_L H_L^* + p_L^*)^{\frac{1}{\rho-1}} \theta_R^{\frac{1}{1-\rho}} T}{w_L^{\frac{\rho}{\rho-1}} + (w_L H_L^* + p_L^*)^{\frac{\rho}{\rho-1}} \theta_R^{\frac{1}{1-\rho}} + 1} \right) + s_L N_L \left(\frac{w_L(w_L H_L^* + p_L^*)^{\frac{1}{\rho-1}} \theta_S^{\frac{1}{1-\rho}} T}{w_L^{\frac{\rho}{\rho-1}} + (w_L H_L^* + p_L^*)^{\frac{\rho}{\rho-1}} \theta_S^{\frac{1}{1-\rho}} + 1} \right) = M_q ,$$

which given proportional supply (9) can be expressed as

$$(1-s_L)g_{L,R}^* + s_L g_{L,S}^* = \frac{M_q}{N_L} = \frac{M}{N}. \quad (33)$$

Similarly, the pricing outlet will clear at the full price $(w_H H_H^* + p_H^*)$ that satisfies

$$(1-s_H)g_{H,R}^* + s_H g_{H,S}^* = \frac{M_p}{N_H} = \frac{M}{N}. \quad (34)$$

Comparing (33) and (34), we see that the consumption of g is equalized *across the average person of each income group*. That is, equality of consumption will hold between income groups, but it is not necessarily the case that $g_{L,R}^* = g_{H,R}^*$ and $g_{L,S}^* = g_{H,S}^*$. Intuitively, the problem is that the effect of preference strength on a person's demand for the target good, $\partial g^* / \partial \theta$ depends on variables such as full price, income, and preference distribution, each of which may differ across outlets. Thus, at the separating equilibrium, there is no reason to believe that individuals with a common θ at different outlets will purchase identical quantities of g^* .

Fortunately, the equalization of average consumption across income groups places substantial constraints on inequality caused by income. In particular, differences in preference strength will dominate differences in income.

Proposition 5 Under the (income) separating equilibrium defined in *Proposition 4*, every individual with θ_S will purchase more g than any individual with θ_R .

Proof: from $\left. \frac{\partial g^*}{\partial \theta} \right|_{P_g=P_g^*} > 0$ and from (33), it follows that $g_{L,S}^* > \frac{M}{N} > g_{L,R}^*$. From

$\left. \frac{\partial g^*}{\partial \theta} \right|_{P_g=P_g^*} > 0$ and from (34) it follows that $g_{H,S}^* > \frac{M}{N} > g_{H,R}^*$. Thus every $g_{i,S}^*$ exceeds

every $g_{i,R}^*$.

Finally, there will again be an equivalent pooled equilibrium at a single outlet that corresponds to our separating equilibrium. As with homogeneous preferences, this occurs at the unique (H^{**}, p^{**}) where the isoprice line for individuals of each income group cross, as defined in (32). The reasoning is analogous to the proof of Proposition 3, and is omitted.

To summarize our results, a policy maker who cannot distinguish high and low wage individuals can use differential time and money pricing of a target good to make its consumption independent of wage (at least on average) yet dependent on strength of preference. Queuing time among high wage individuals can be set to zero. Queuing time among low wage individuals is bounded below at the time price H^{**} at which isoprice lines. If this queuing time were sufficiently high that the value of the good (θ) fell during the wait, the normative appeal of our separating equilibria might diminish, as g received now would be unequal to g received later. A policy maker persevering with the mechanism might then have to set p_L^* and p_H^* so as to limit the difference in queuing times so as to convince the poor to remain at the queuing outlet.¹⁴

3.5 Multiple outlets

Thus far, our analysis has been restricted to binary classifications of income and preference type. However, our results could readily be extended in a discrete

framework to any countable finite number of wage levels and preference strengths. Fig. 5, for example, illustrates how consumption could be equalized for four income groups ($w_1 < w_2 < w_3 < w_4$) when resale is infeasible, preferences are homogeneous and $\rho > 0$. The policy maker could use four outlets with respective money prices set within the ranges ($p_1 < p_2 < p_3 < p_4$). The money/time price pairs A , B , C and D provide a sample separating equilibrium.¹⁵ Using reasoning analogous to that for existence in Appendix 1, members of each wage group would find the full price lowest in their own outlet. Alternatively, the policy maker could achieve equivalent allocations using two pooled outlets with unique money prices ($p_{1,2}^{**} < p_{3,4}^{**}$). More generally, it is anticipated that K income groups could be accommodated with K separating outlets, or with the largest integer lower than or equal to $K/2$ pooled outlets.

4. Conclusion

In this paper, we have considered the problem of a policy maker with the in-kind distributional objective of ‘specific egalitarianism,’ or that access to a target good be made independent of income, but increasing in preference or need. This objective could result from good-specific distributional concerns in society at large, and the inability of the policy maker to distinguish high and low ability individuals in the standard tax and transfer system.

We described circumstances under which this objective could be achieved by making the good simultaneously available at parallel outlets charging different money and time prices. In particular, with two income groups, a policy maker could allocate a fixed supply of the target good at two outlets in proportion to the income distribution, set a high money price at one, and a low (even negative) money price at the other. Queuing time would equilibrate at each outlet. Those with a high wage would choose the outlet with a high money price and little or no waiting, and those

Fig. 5. Isoprice lines for four wage groups ($\rho > 0$)

with a low wage would choose the outlet with low money price and a substantial queue. Alternatively, the policy maker could induce an equivalent pooled equilibrium in which both income groups would be served at a single outlet. This equilibrium would feature a unique money and time price that each income group would find equivalent to the full price they would face at the separating equilibria.

We find that when resale is infeasible and preferences are homogeneous, our proposal exactly equalizes consumption of the target good across rich and poor. When preferences for the target good are heterogeneous, differential pricing equalizes the average consumption of the target good across income groups. At the individual level, those with a higher strength of preference for the good will always receive more of it than those with a lower strength, regardless of income. These results will hold so long as separating equilibria exist, which requires that the distribution of preferences for the target good not differ excessively between the rich and the poor.

While differential pricing involves the allocative inefficiency of optional queuing, it has the advantage over uniform disbursement that differences in relative

preference orderings are respected. The policy maker could minimize queuing time by setting separate money prices such that high wage individuals would pay purely by money, and low wage individuals would pay the highest money price consistent with keeping high wage individuals out of their outlet.

The informational requirements of differential allocation are modest; the policy maker must know a population's income and preference distribution, but not an individual's. Even an exact knowledge of the preference distribution is not essential for separating equilibria, because the policy maker can set each outlet's money price within a range and achieve the same distribution of the target good.

Unfortunately, our mechanism suffers from several limitations. First, as Nichols et al. (1971) observed, the existence of non-labour income raises the possibility that wealthy retirees might choose outlets targeted to the poor. Second, if resale of the target good is feasible, all potential separating equilibria created by the mechanism are vulnerable because the full price faced by the poor is less than that faced by the rich. To prevent resale of the target good from poor to rich, the policy maker would need to purposefully raise the transactions costs of resale, either to low wage sellers or high wage buyers. This could be achieved by personalizing the purchase of the good with purchaser details that require verification with use.

Third, if the minimum queuing time for low wage individuals were substantial enough that the value of the target good to those individuals depreciated during the queue, the consumption of the rich would not exceed that of the poor in quantity, but it surely would in value. Lastly, our proposal requires the policy maker to bear the expense and complexity of administering multiple outlets. We note that real world target goods such as health care, public ferry tickets, hiking permits, postal services, and immigration processing offer at most a few price/time combinations.

Nonetheless, with judicious money pricing, even a few outlet choices will greatly diminish the disparity of income of individuals per outlet, and the inequality of consumption that results.

Appendix 1

Proof of Existence and Uniqueness of a Separating Equilibrium

Suppose the policy maker adopts proportional allocation across two outlets as in (9), and sets p_L at a ‘queuing’ outlet and p_H at a ‘pricing’ outlet according to $(p_{L,\min}^* \leq p_L \leq p^{**})$ and $(p^{**} \leq p_H \leq p_{pH}^*)$ such that $p_H \neq p_L$, where p^{**} is defined by (11).

We claim that any money price pair (p_L, p_H) as chosen above will result in the separation of high and low wage individuals to their respective outlets, and therefore a unique set of full prices $P_g^*|_{w_H}$, $P_g^*|_{w_L}$ and resulting allocation of l_H^*, y_H^*, g_H^* and l_L^*, y_L^*, g_L^* and utilities.

Suppose all the poor go to the queuing outlet and all the rich go to the pricing outlet. As was shown by the signing of the partial derivatives in (5) and (6), the equilibrium isoprice line of the rich must cut the equilibrium isoprice line of the poor from above. This is repeated for ease of reference in Fig. A.1 below.

Thus, if a single member of N_L switched to the pricing outlet, he would face a full price

$$w_L H_H^* + p_H^* > w_L H_H^* + \cancel{p} = w_L H_L^* + p_L^* \quad (\text{A.1})$$

The inequality in (A.1) follows from hypothetically comparing the lower money price $\cancel{p}$ that would be needed to clear the pricing outlet with queuing time held at H_H^* if its inhabitants had their wage cut from w_H to w_L , all else controlled. The equality in (A.1) follows because $(H_H^*, \cancel{p})$ is on the same isoprice line for low wage people as (H_L^*, p_L^*) . Taken together, the two relations of (A.1) show that any low wage individual faces a higher full price by deviating from the queuing outlet.

Similarly, if a single member of N_H switched to the queuing outlet, he would face a full price

$$w_H H_L^* + p_L^* > w_H \cancel{H} + \cancel{p}_L = w_H H_H^* + p_H^* \quad (\text{A.2})$$

Again, the inequality in (A.2) follows from hypothetically comparing the lower time price $\cancel{H}$ that would be needed to clear the queuing outlet with money price fixed at p_L^* if its inhabitants had their wage raised from w_L to w_H , all else controlled. The equality in (A.2) follows because $(\cancel{H}, p_L^*)$ is on the same isoprice line for a high wage

Fig. A1. Isoprice lines for two wage levels, $\rho > 0$

individual as (H_H^*, p_H^*) . Taken together, the two relations of (A.2) show that any high wage individual faces a higher full price by deviating from the pricing outlet.

Appendix 2

Proof of the effect of an increase in wage disparity on the intersection of isoprice lines.

Consider three wage levels $w_L < w_M < w_H$. Next define $(H_{L,M}^{**}, p_{L,M}^{**})$ as the intersection point of the w_L and w_M isoprice lines, $(H_{L,H}^{**}, p_{L,H}^{**})$ as the intersection point of the w_L and w_H isoprice lines, and $(H_{M,H}^{**}, p_{M,H}^{**})$ as the intersection point of the w_M and w_H isoprice lines.

Substituting out the common $p_{L,H}^{**}$ for the w_L and w_H isoprice lines,

$$H_{L,H}^{**} = \frac{P_g^*|_{w_H} - P_g^*|_{w_L}}{w_H - w_L}. \quad (\text{A.2.1})$$

This can be expanded to be expressed as

$$H_{L,H}^{**} = \frac{P_g^*|_{w_H} - P_g^*|_{w_M}}{w_H - w_M} \frac{w_H - w_M}{w_H - w_L} + \frac{P_g^*|_{w_M} - P_g^*|_{w_L}}{w_M - w_L} \frac{w_M - w_L}{w_H - w_L}, \quad (\text{A.2.2})$$

or as

$$H_{L,H}^{**} = H_{M,H}^{**} \pi + H_{L,M}^{**} (1 - \pi) \quad \text{where } 0 < \pi < 1. \quad (\text{A.2.3})$$

If $H_{M,H}^{**} < H_{L,M}^{**}$, (A.2.3) implies that $H_{M,H}^{**} < H_{L,H}^{**} < H_{L,M}^{**}$, or that the w_L isoprice line would cross the w_H isoprice line before it crosses the w_M isoprice line as is illustrated in Figs. A.2 and A.3 below. Conversely, if $H_{M,H}^{**} > H_{L,M}^{**}$, (A.2.3) would imply that $H_{L,M}^{**} < H_{L,H}^{**} < H_{M,H}^{**}$, or that the w_L isoprice line would cross the w_M isoprice line before it crosses the w_H isoprice line.

We will show that $H_{M,H}^{**} < H_{L,M}^{**}$. First, note from (A.2.1) that $H_{L,H}^{**} = \frac{\Delta P_g^*}{\Delta w}$, or the

slope of the P_g^* function with respect to w . Thus if $\frac{\partial^2 P_g^*}{\partial w^2} < 0$ then $H_{M,H}^{**} < H_{L,M}^{**}$.

Totally differentiating the market clearing condition (4) with g^* defined as in (3),

$$\frac{\partial P_g^*}{\partial w} = - \frac{\frac{\partial g^*}{\partial w}}{\frac{\partial P_g^*}{\partial g^*}} = \frac{P_g^{\frac{1}{\rho-1}} \theta^{\frac{1}{1-\rho}} T \left(\frac{1}{1-\rho} w^{\frac{\rho}{\rho-1}} + 1 \right) + P_g^{\frac{\rho+1}{\rho-1}} \theta^{\frac{2}{1-\rho}} T}{\frac{1}{1-\rho} w P_g^{\frac{2-\rho}{\rho-1}} \theta^{\frac{1}{1-\rho}} T (w^{\frac{\rho}{\rho-1}} + 1) + w P_g^{\frac{2}{\rho-1}} \theta^{\frac{2}{1-\rho}} T} \equiv \frac{A}{B} \quad (\text{A.2.4})$$

Fig. A.2 Feasible Range of Money Prices as w_M increases to w_H ($\rho > 0$)

Fig. A.3 Feasible Range of Money Prices as w_M increases to w_H ($\rho < 0$)

It follows that

$$\frac{\partial^2 P_g^*}{\partial w^2} = \frac{\frac{\partial A}{\partial w} B - \frac{\partial B}{\partial w} A}{B^2} \quad (\text{A.2.5})$$

The denominator of (A.2.5) is positive. The numerator is

$$-\frac{1}{(1-\rho)^2} P_g^{\frac{3-\rho}{\rho-1}} \theta^{\frac{2}{\rho-1}} w^{\frac{2\rho}{\rho-1}} T^2 - \frac{2}{(1-\rho)} P_g^{\frac{3}{\rho-1}} \theta^{\frac{3}{1-\rho}} w^{\frac{\rho}{\rho-1}} T^2 + \frac{-2(\rho-1)^2 - \rho}{(1-\rho)^3} P_g^{\frac{3-\rho}{\rho-1}} \theta^{\frac{2}{1-\rho}} w^{\frac{\rho}{\rho-1}} T^2$$

$$-\frac{1}{1-\rho}P_g^{\frac{3}{\rho-1}}\theta^{\frac{3}{1-\rho}}T^2 - \frac{1}{1-\rho}P_g^{\frac{3-\rho}{\rho-1}}\theta^{\frac{2}{1-\rho}}T^2 - P_g^{\frac{3+\rho}{\rho-1}}\theta^{\frac{4}{1-\rho}}T^2 - P_g^{\frac{3}{\rho-1}}\theta^{\frac{3}{1-\rho}}T^2 \quad (\text{A.2.6})$$

(A.2.6) is negative for all values of $\rho < 1$.

The fact that an increase in w from w_M to w_H lowers H^{**} and raises p^{**} for positive or negative values of ρ , and moves the lower bound of the high wage isoprice line $(H_{H,\max}^*, p_{H,\min}^*)$ further to the south-west, together with the signs of (5) and (6), ensures that the potential range of money prices that can be set by the policy maker will increase.

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¹ Clark and Kim (2005) compare social welfare under the 'pay or wait' mechanism with a proportional tax and uniform allocation system that also weakly achieves specific egalitarianism (SE). This paper finds that SE may be achieved more efficiently under the pay or wait mechanism than tax and uniform provision as (i) the relative importance of the target good rises, (ii) the elasticity of substitution between goods falls, (iii) variation in preferences increases and (iv) income inequality rises or the proportion of the poor falls.

² Preferences are more general than in Alexeev and Leitzel (2001), who assume equal weight Cobb Douglas preferences between a single good and leisure. We are less general, however, than O'Shaughnessy (2000), who assumes general concave utility over a single good and leisure.

³ More precisely, $\rho = (\sigma - 1) / \sigma$, where σ is the Allen elasticity of substitution between goods.

⁴ This assumes that individuals must queue once per unit purchased, and that everyone in a given outlet will wait an identical period of time per unit purchased. This is a common reduced form way of modeling that queuing time increases with demand (Barzel, 1974; Sah, 1987; Suen, 1989; Polterovich, 1993; O'Shaughnessy, 2000; and Alexeev and Leitzel, 2001). Alternatives have been proposed, such as queuing time depending on show-up time (Holt and Sherman, 1982), or fixed time costs for any quantity of purchase (Weitzman, 1991).

⁵ An alternative would be to extend the approach of Alexeev and Leitzel (2001), and assume 1) identical constant returns to scale production of y and g , 2) that workers are paid the value of their marginal product, and 3) that the government is the sole purchaser of all g produced. This would result in a labour allocation between the two sectors that is under-identified, and would add little to our distributional results. Readers interested in the efficiency aspects of a general equilibrium approach are referred to Clark and Kim (2005).

⁶ Existence and uniqueness are satisfied in that the excess demand function (4) is monotonically decreasing in full price, takes on a negative value at an infinite price, and takes on an infinite value at zero price. The same conditions are satisfied with multiple outlets and heterogeneous preferences later in the paper.

⁷ Wage differentials are exogenous here, but could reflect differences in worker marginal product.

⁸ p_{pL}^* is similarly defined as the money price that would clear the outlet targeted to the poor with zero queuing.

⁹ A detailed proof of this claim is given in Appendix 2. It should be noted that at the borderline case of Cobb-Douglas elasticity of substitution ($\rho = 0$), an increase in w_H would pivot the high wage isoprice line around an H^{**}, p^{**} pair fixed on the horizontal axis where $p^{**} = 0$. For the high wage group the range of possible money prices would still increase, but for the low wage group it would remain at $(p_{L,\min}^* < p_L < p^{**})$.

¹⁰ The authors are grateful to a referee for pointing out this potential problem.

¹¹ Setting time rather than money prices ensures that the money price at the pricing outlet can adjust to fully clear the outlet when demand there drops in response to the resale market. Setting the time price at the pricing outlet specifically at zero avoids the problem of identifying the maximum possible value $H_{H,\max}^*$ that ensures that low wage individuals would prefer the queuing to the pricing outlet at black market equilibrium.

¹² Weitzman (1977) adopts the more precise ideal of a distribution of the target good that would result if total actual income were equally distributed but tastes differed, in a market with pure pricing.

¹³ That is, individuals with a stronger demand for g would desire to supply negative labour and “purchase g for a living” at a smaller per unit subsidy p than individuals with a regular demand for g .

Formally, with the full price of g constant, the partial derivative of $p_{\min}^* = -\theta^{\frac{1}{\rho-1}} (P_g^*)^{\frac{1}{1-\rho}}$ with respect to θ is positive. Since both θ types face the same $P_g^* (= wH_{\max}^* + p_{\min}^*)$, an increase in θ that raises $p_{\min}^*$ must also decrease $H_{\max}^*$.

¹⁴ If θ were replaced by $\theta\delta^H$, where $0 < \delta < 1$, an individual will choose an outlet based not only on the lowest full price, but also the resulting value of the target good. This would re-enforce the rich in their choice of the pricing outlet, but provide a potential incentive for the poor to also switch to the pricing outlet if the resulting utility gain from the queuing time reduction exceeds the utility loss from the higher full price. Preserving separation could require the policy maker to raise p_L^* so as to approach p^{**} from below, and/or lower p_H^* so as to approach p^{**} from above.

¹⁵ The isoprice lines would intersect as shown in Fig. 4 from the same reasoning given in Appendix 2. Considering the bottom three wage groups, Appendix 2 shows that $H_{2,3}^{**} < H_{1,3}^{**} < H_{1,2}^{**}$, or that the w_1 isoprice line would cross the w_3 isoprice line before it crosses the w_2 isoprice line. Applying the same reasoning to the top three wage groups and combining yields $H_{3,4}^{**} < H_{2,4}^{**} < H_{2,3}^{**} < H_{1,3}^{**} < H_{1,2}^{**}$.